

NOTES ON INVENTORY MANAGEMENT & CONTROL

Inventory is the stock of any item or resource used in an organization. Inventory includes: raw materials, finished products, component parts, supplies, and work-in-process. An inventory system is the set of policies and controls that monitors levels of inventory and determines what levels should be maintained, when stock should be replenished, and how large orders should be.

The purposes of inventories are:

1. To maintain independence of operations
2. To meet variation in product demand
3. To allow flexibility in production scheduling
4. To provide a safeguard for variation in raw material delivery time
5. To take advantage of economic purchase order size

INVENTORY COSTS

Five types of costs need to be considered when analyzing inventory decisions:

1. Holding (or carrying) costs: storage facilities, handling, insurance, pilferage, breakage, obsolescence, depreciation, taxes, and the opportunity cost of capital.
2. Setup (or production change) costs: line conversion, equipment change-over, report preparation, etc.
3. Ordering costs: typing, calling, transportation, receiving, etc. This cost does not depend or vary on the number ordered.
4. Shortage costs (stockout costs): the loss due to losing a specific sale, customers' goodwill, or future business.
5. Cost of the item

INDEPENDENT VERSUS DEPENDENT DEMAND

Independent demand (i.e., the demand by consumers) is influenced by market conditions outside the control of operations. Independent demand calls for a replenishment philosophy. Orders are made to replenish inventory.

Dependent (or derived) demand is related to the demand for another item. For example, parts, intermediate goods, and raw materials face a demand dependent on the demand for the final goods. Dependent demand calls for a requirements philosophy. Orders are made if there is a demand or requirement for the final product.

ECONOMIC ORDER QUANTITY (EOQ)

The economic order quantity or EOQ is the certain amount to be ordered at specific intervals. It gives the perfect sawtooth pattern in a graph of inventory versus time.

EOQ is simple to understand and use but it has several restrictive assumptions which are also disadvantages in practice. Even with these weaknesses, EOQ is a good place to start to understand inventory systems. EOQ assumes:

1. Demand rate is constant, uniform, recurring, and known.
2. Lead time is constant and known.
3. Price per unit of product is constant; no discounts are given for large orders.
4. Inventory holding cost is based on average inventory.
5. Ordering or setup costs are constant.
6. All demands will be satisfied; no stockouts are allowed.

D = demand rate, units per year

S = cost per order placed, or setup cost, dollars per order

C = unit cost, dollars per unit

i = holding or carrying rate, percent of dollar value per year

Q = lot size, units

TC = total or ordering cost plus carrying cost, dollars per year

Annual purchase cost = DC

Annual ordering cost = $(D/Q)S$

Annual holding cost per year = $HQ/2 = iCQ/2$

TC = total annual cost = $DC + (D/Q)S + iCQ/2$

Solving for minimum TC yields the economic order quantity:

$$EOQ = \sqrt{2SD / iC}$$

Number of orders per year = D/Q

Number of months between orders = $12 / (D/Q)$

A BASIC EOQ EXAMPLE

A grocery store sells 10 cases of coffee each week. Each case costs \$80. The cost of ordering is \$10 per order. Holding or carrying cost is estimated to be 30% of the inventory value per year. So the variables are defined as:

$$\begin{aligned}D &= 520 \text{ cases/year} = 10 \text{ cases/week} * 52 \text{ weeks/year} \\S &= \$10 \text{ per order} \\i &= 30\% \text{ (or } 0.30) \\C &= \$80 \text{ per case}\end{aligned}$$

$$EOQ = \sqrt{\frac{2SD}{iC}} = \sqrt{\frac{2*10*520}{0.3*80}} = 20.8 \approx 21 \text{ cases per order}$$

How often is the coffee ordered?

$$520/21 \approx 25 \text{ orders per year. Or every 15 days (} 365/25 \approx 15 \text{)}$$

The total cost of ordering and carrying inventory is:

$$TC = S(D/Q) + iCQ/2 = 10*(520/21) + 0.3*80*21/2 = \$500 \text{ per year}$$

If demand increased by 50% to 15 cases per week:

EOQ increases to 25 cases per order (from 21) which, please note, is NOT a 50% increase. The ordering interval drops to every 12 days. TC rises to \$612.

If carrying cost was 45% (a 50% rise): (Demand is at the original 10 cases per week.)

EOQ drops to 17. The ordering interval drops to 12. TC rises to \$612.

Using EOQ for Determining Lot Size

An ice cream company produces 100,000 cases of black walnut & cherry ice cream each year. Switching the production line from another flavor to black walnut & cherry costs \$1,000 each time for cleaning, assembling, putting the ingredients in the correct places, etc. The cost of each case is \$100. The company estimates their inventory cost is 30% of inventory value per year.

$$D = 100,000 \text{ cases/year}$$

$$S = \$1000 \text{ per setup}$$

$$i = 30\% \text{ (or } 0.30\text{)}$$

$$C = \$100 \text{ per case}$$

$$EOQ = \sqrt{\frac{2SD}{iC}} = \sqrt{\frac{2 \cdot 1000 \cdot 100000}{0.3 \cdot 100}} = 2,582 \text{ cases per production run}$$

How many lots or runs per year?

$$100000 / 2582 = 38.7 \text{ orders per year}$$

The total cost of setup and carrying inventory is:

$$TC = S(D/Q) + iCQ/2 = 1000 \cdot (100000/2582) + 0.3 \cdot 100 \cdot 2582/2 = \$77,460 \text{ per year}$$

I doubt that the company would be so specific as to order a run of 2,582 cases. If they decided to use orders of 2,500 cases per run, their total cost would be:

$$TC = S(D/Q) + iCQ/2 = 1000 \cdot (100000/2500) + 0.3 \cdot 100 \cdot 2500/2 = \$77,500 \text{ per year}$$

For an increase of \$40 per year, I suspect the company may decide that the round number of 2,500 may be easier to use on the production floor and elsewhere in the office.

FIXED-ORDER QUANTITY MODELS (or Q-models)

Fixed-order quantity models attempt to determine the specific point, R, at which an order will be placed and the size of that order, Q. The fixed-order quantity is also called the Q-system since the order quantity Q is fixed.

With a fixed-order quantity system, an order of size Q is placed when the inventory available (currently in stock and on order) reaches the point R.

R is determined as the average demand over the lead time (that is, the time between ordering and receiving) plus a safety stock to reflect variation in demand over time.

R = reorder point

\bar{d} = average daily demand (constant)

L = lead time in days (constant)

z = safety factor (number of standard deviations for a specified service probability)

S_L = standard deviation of demand over the lead time

$$R = \bar{d}L + zS_L$$

Service level is the percentage of customer demands satisfied from inventory.

Shortage or stockout percentage = 100 - service level.

EXAMPLE USING CONTINUOUS REVIEW WITH FIXED-ORDER

QUANTITIES (also called perpetual review or the Q-system, since the order quantity remains the same)

A repair shop sells tires and has assembled the following information for one specific type:

D = 500 tires/year

S = \$40 per order

i = 25% (or 0.25)

C = \$40 per tire

L = 4 days (250 working days per year)

S = s.d. of daily demand = 1 tire

desired service level = 95%. So, z = 1.65

$$EOQ = \sqrt{\frac{2SD}{iC}} = \sqrt{\frac{2 \cdot 40 \cdot 500}{0.25 \cdot 40}} = 63.25 \text{ tires per order}$$

$$\text{Reorder Point} = ROP = m + s = 8 + 3.3 = 11.3 \approx 12 \text{ tires}$$

m = average demand over lead time = average demand per working day * lead time

$$= \bar{d}L = (500/250) * 4 = 8 \text{ tires}$$

$$s = \text{safety stock} = zS_L = 1.65 * 1 * \sqrt{4} = 3.3 \text{ tires}$$

The specific Q-rule is: Continuously review stock level and order 63 tires when stock reaches the reorder point of 12 tires.

EOQ with Price Discounts

A supplier of cleaning products offers a supermarket the following deal. If the supermarket buys 29 cases or less, the cost is \$25 per case. If they buy 30 or more cases, the cost will be \$20 per case. The supermarket's ordering cost is \$20 per order and the carrying cost is 15 percent per year. The supermarket sells 50 cases per year.

Procedure when analyzing the EOQ with discounts:

1. Calculate the EOQ for the lowest cost per unit. If the EOQ is above the price break, this is the most economical quantity.
2. If the EOQ in 1 above is below the price break, use the next lowest price to calculate the EOQ. Continue calculating EOQs until a feasible EOQ is found.
3. Calculate the total cost of the EOQ, if found, and the total cost of the higher price breaks.
4. The minimum of these total costs indicates the most economic order quantity.

FIXED-TIME PERIOD MODELS (or P-Models)

In a fixed-time period system, inventory is counted only at particular times and the size of the order varies. Compared to the Q-system, the P-system, does not have a reorder point but rather a target inventory. The P-system does not have an economic order quantity since it varies according to demand. The P-system requires a larger safety stock for the same service level.

The P system is determined by two parameters.

P = the time between orders. We can approximate P as $Q/D = \sqrt{2 \cdot S / iCD}$

T = target inventory level = $m' + s'$

m' = average demand over $P + L$

$s' = z \cdot \text{std. dev.}(P+L)$

L = lead time in days (between placing and receiving an order)

z = safety factor (number of standard deviations for a specified service probability)

EXAMPLE USING PERIODIC REVIEW (also called the fixed order interval (FOI) or the

P-system, since the time period between orders remains the same)

If the same repair shop (as above) orders tires at specific intervals (say once a month or every 21 working days on average), the inventory orders are done differently. The EOQ is still the same (63.25 tires per order), but we now talk about the target inventory.

Each month's order = target inventory - stock on hand on day of order

$$\text{Target inventory} = m' + s' = 50 + 8.25 = 58.25 \approx 59 \text{ tires}$$

$$m' = (500/250) * (21+4) = 50$$

$$s' = Z * S_{P+L} = 1.65 * 1 * \text{sqrt}(21+4) = 8.25$$

The specific P-rule is: Review the stock on, say, the first working day of every month and order the difference between the inventory on hand on that day and the target inventory of 59 tires.

USING P AND Q SYSTEMS IN PRACTICE

The choice between P and Q is not simple, but there are some conditions under which the P system may be preferred over the Q system.

1. When orders must be placed or delivered at specified intervals (e.g., canned goods in a grocery store)
2. When multiple items are ordered from the same supplier and delivered in the same shipment (e.g., different flavors of ice cream)
3. For inexpensive items which are not maintained on perpetual inventory records (e.g., nuts and bolts in a bin)

The P-system requires less record keeping, but a larger safety stock. So the Q-system may be preferred for expensive items to keep carrying costs lower.

OTHER SYSTEMS

Optional replenishment system: Reviewing on a fixed frequency and ordering a specified quantity if inventory is below a certain level. This is a mix of the P and Q systems.

Two-bin system: An amount equal to R is kept in reserve in a "second" bin. When the first bin is emptied, the second bin is emptied into the first and an order of size Q is placed.

One-bin system: This is the P-system where the one bin is reviewed at a fixed interval and inventory brought up to a certain inventory level.

ABC Inventory planning: Inventory items are classified into three groups on the basis of annual dollar volume. More attention is given to those inventory items with a high dollar volume

compared to a low dollar volume.

- A: high dollar volume items, say 15% of the total number of items
- B: moderate dollar volume items, say the next 35%
- C: low dollar volume items, the last 50%

Summary Problem

The local cooperative supply company stocks a vitamin/mineral premix for a specialized poultry feed. This premix has a demand of 4 units per month and a cost of \$25 per package. The carrying charge is 20 percent per year and the ordering cost is \$15 per order.

- a. What is the EOQ for this item?
- b. How often will the dealer reorder this premix?
- c. What is the annual ordering and carrying cost of the premix?
- d. The cooperative is considering installing either a Q or P system for inventory control. The past standard deviation of demand for the premix has been 2.5 units per month and the replenishment lead time is two months. A 95 percent service level is desired.
 - 1. If a continuous review system is used, what is the value of Q and R that would be applicable?
 - 2. If a periodic review system is used, what is the value of P and T that would be applicable?
 - 3. What are the pros and cons of using the P system compared to the Q system for this premix?