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Financial management

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Undergraduate study in
Economics, Management,
Finance and the Social Sciences

This is an extract from a subject guide for an undergraduate course offered as part of the University of London International Programmes in Economics, Management, Finance and the Social Sciences. Materials for these programmes are developed by academics at the London School of Economics and Political Science (LSE).

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LSE THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

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It is a revised edition of previous editions of the guide prepared by J. Dahya and R.E.V. Groves, and draws on the work of those authors.

This is one of a series of subject guides published by the University. We regret that due to pressure of work the author is unable to enter into any correspondence relating to, or arising from, the guide. If you have any comments on this subject guide, favourable or unfavourable, please use the form at the back of this guide.

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Introduction

59 Financial management is a 300 course offered on the degrees and diplomas in Economics, Management, Finance and the Social Sciences (EMFSS) suite of programmes awarded by the University of London International Programmes.

Financial management is part of the decision-making, planning and control subsystems of an enterprise. It incorporates the:

- treasury function, which includes the management of working capital and the implications arising from exchange rate mechanisms due to international competition
- evaluation, selection, management and control of new capital investment opportunities
- raising and management of the long-term financing of an entity
- need to understand the scope and effects of the capital markets for a company, and
- need to understand the strategic planning processes necessary to manage the long and short-term financial activities of a firm.

The management of risk in the different aspects of the financial activities undertaken is also addressed.

Studying this course should provide you with an overview of the problems facing a financial manager in the commercial world. It will introduce you to the concepts and theories of corporate finance that underlie the techniques that are offered as aids for the understanding, evaluation and resolution of financial managers' problems.

This subject guide is written to supplement the Essential and Further reading listed for this course, not to replace them. It makes no assumptions about prior knowledge other than that you have completed **25 Principles of accounting**. The aim of the course is to provide an understanding and awareness of both the underlying concepts and practical application of the basics of financial management. The subject guide and the readings should also help to build in your mind the ability to make critical judgments of the strengths and weaknesses of the theories, just as it should be helping to build a critical appreciation of the uses and limitations of the same theories and their possible applications.

Aims and objectives

This course aims to cover the basic building blocks of financial management that are of primary concern to corporate managers, and all the considerations needed to make financial decisions both inside and outside firms.

This course also builds on the concept of net present value and addresses capital budgeting aspects of investment decisions. Time value of money is then applied to value financial assets, before extensively considering the relationship between risk and return. This course also introduces the theory and practice of financing and dividend decisions, cash and working capital management and risk management. Business valuation and mergers and acquisitions will also be discussed.

By the end of this course and having completed the Essential reading and activities, you should be able to:

Subject-specific objectives

- describe how different financial markets function
- estimate the value of different financial instruments (including stocks and bonds)
- make capital budgeting decisions under both certainty and uncertainty
- apply the capital assets pricing model in practical scenarios
- discuss the capital structure theory and dividend policy of a firm
- estimate the value of derivatives and advise management how to use derivatives in risk management and capital budgeting
- describe and assess how companies manage working capital and short-term financing
- discuss the main motives and implications of mergers and acquisitions.

Intellectual objectives

- integrate subject matter studied on related modules and to demonstrate the multi-disciplinary aspect of practical financial management problems
- use academic theory and research to question established financial theories.

Practical objectives

- be more proficient in researching materials on the internet and Online Library
- be able to use Excel for statistical analysis.

Syllabus

The subject guide examines the key theoretical and practical issues relating to financial management. The topics to be covered in this subject guide are organised into the following 12 chapters:

Chapter 1: Financial management function and environment

This chapter outlines the fundamental concepts in financial management and deals with the problems of shareholders' wealth maximisation and agency conflicts.

Chapter 2: Investment appraisals

In this chapter we begin with a revision of investment appraisal techniques. The main focus of this chapter is to examine the advantages of using the discounted cash flow technique and its application in complex investment scenarios: capital rationing, replacement decision, project deferment and sensitivity analysis.

Chapter 3: Risk and return

We formally examine the concept and measurement of risk and return in this chapter. We also look at the necessary conditions for risk diversification, portfolio theory and the two fund separation theorem. Asset pricing models are discussed and practical considerations in estimating beta will be covered. Empirical evidence for and against the asset pricing models will also be illustrated.

Chapter 4: Capital market efficiency

This chapter discusses the concepts and implications of market efficiency and the mechanism of equity and debt issuance.

Chapter 5: Sources of finance

In this chapter we focus on how companies raise funds from both the stock and bond markets and discuss the advantages and disadvantages of each type of financing method.

Chapter 6: Capital structure

This chapter critically reviews the existing leading theories of capital structure. Specifically, the trade-off theory, signalling effect, agency cost of equity and debt and the pecking order theory will be examined. We will also evaluate the practical considerations of capital structure decisions made by corporate managers.

Chapter 7: Dividend policy

This chapter aims to explore how the amount of dividend paid by corporations would affect their market values. The tax, signalling and agency effects of dividend will be discussed.

Chapter 8: Cost of capital and capital investments

In this chapter we discuss how the cost of capital can be adjusted when firms are financed with a mixture of debt and equity.

Chapter 9: Valuation of business

We introduce the valuation of equity, debt, convertibles and warrants in this chapter.

Chapter 10: Mergers

This chapter focuses on the theory and motives of mergers and acquisitions. The determination of merger value and the defensive tactics against merger threats will also be covered. The empirical evidence of using financial ratios to predict mergers and acquisitions will be discussed.

Chapter 11: Financial planning and working capital management

The importance of managing cash and short-term financing will be discussed in this chapter.

Chapter 12: Risk management

This chapter provides an introduction to risk management including: hedging, futures, options and derivatives and their uses in both long-term and short-term situations.

Reading

Essential reading

Brealey, R.A., S.C. Myers and F. Allen *Principles of corporate finance*. (New York: McGraw-Hill, 2010) tenth edition [ISBN 9780071314268]. Hereafter called BMA, this textbook deals with most of the topics covered in this subject guide.

Detailed reading references in this subject guide refer to the edition of the set textbook listed above. New editions of this textbook may have been published by the time you study this course. You can use a more recent edition of this book or of any of the books listed below; use the detailed chapter and section headings and the index to identify relevant readings. Also check the VLE regularly for updated guidance on readings.

Further reading

Please note that as long as you read the Essential reading you are then free to read around the subject area in any text, paper or online resource. You will need to support your learning by reading as widely as possible and by thinking about how these principles apply in the real world. To help you read extensively, you have free access to the virtual learning environment (VLE) and the University of London Online Library (see below).

Other useful texts for this course include:

Arnold, G. *Corporate financial management*. (Harlow: Financial Times/Prentice Hall, 2008) fourth edition [ISBN 9780273719069]. Hereafter called ARD, this textbook also covers most of the topics in this subject guide. It is less technical than BMA.

Copeland, T.E., J.F. Weston and K.S. Shastri *Financial theory and corporate policy*. (Harlow: Pearson-Addison Wesley, 2004) fourth edition [ISBN 9780321127211].This is a classic finance textbook pitched at an advanced level. You may use this textbook for reference as it contains some useful updates of empirical studies in the field of corporate finance.

Watson, D. and A. Head *Corporate finance passnotes*. (Harlow: Pearson Education, 2010) first edition [ISBN 9780273725268].This concise version of a passnote neatly summarises the key concepts in financial management. You might find it useful as a revision tool.

Apart from the above textbooks, this subject guide also refers to some of the original articles from which the financial management theories are developing. You should refer to the works cited in each chapter for the full reference of these articles.

How to use the subject guide

This subject guide is meant to supplement but not to replace the main textbook. You should use it as a guide to devise a plan for your own study of this subject. Suggested here is one approach in how to use this subject guide.

Approach financial management in the same order as the chapters in this subject guide. It is specifically designed to help you build up your understanding of the subject.

1. For each chapter (apart from this Introduction) you should familiarise yourself with the aim and outcomes before reading the materials.
2. Read the introductory section of each chapter to identify the areas you need to focus on.
3. Carefully read the suggested chapters in BMA, with the aim of gaining an initial understanding of the topics.
4. Read the remainder of the chapter in the subject guide. You may then approach the Further reading suggested in the subject guide and BMA.
5. The subject guide is designed to set the scope of your studies of this topic as well as attempting to reinforce the basic messages set out in BMA. Therefore you should pay careful attention to the examples in both the texts and the subject guide to ensure you achieve that basic understanding. By taking notes from BMA, and then from other books you should have obtained the necessary material for your understanding, application and later revision.
6. Pay particular attention to the practice questions and the examples given in the subject guide. The material covered in the examples and in the activity exercises complements the textbook and is important in your preparation for the examination.

7. Ensure you have achieved the listed learning outcomes.
8. Attempt the Sample examination questions at the end of each chapter and the quizzes on the virtual learning environment (VLE).
9. Check you have mastered each topic before moving on to the next.
10. At the end of your preparations, attempt the questions in the Sample examination paper at the end of the subject guide. Then compare your answers with the suggested solutions, but do remember that they may well include more information than the Examiner would expect in an examination paper, since the guide is trying to cover all possible angles in the answer, a luxury you do not usually have time for in an examination.

Online study resources

In addition to the subject guide and the Essential reading, it is crucial that you take advantage of the study resources that are available online for this course, including the VLE and the Online Library.

You can access the VLE, the Online Library and your University of London email account via the Student Portal at:
http://my.londoninternational.ac.uk

You should have received your login details for the Student Portal with your official offer, which was emailed to the address that you gave on your application form. You have probably already logged in to the Student Portal in order to register! As soon as you registered, you will automatically have been granted access to the VLE, Online Library and your fully functional University of London email account.

If you forget your login details at any point, please email uolia.support@london.ac.uk quoting your student number.

The VLE

The VLE, which complements this subject guide, has been designed to enhance your learning experience, providing additional support and a sense of community. It forms an important part of your study experience with the University of London and you should access it regularly.

The VLE provides a range of resources for EMFSS courses:

- Self-testing activities: Allow you to test your own understanding of subject material.
- Electronic study materials: The printed materials that you receive from the University of London are available to download, including updated reading lists and references.
- Past examination papers and *Examiners' commentaries*: These provide advice on how each examination question might best be answered.
- A student discussion forum: This is an open space for you to discuss interests and experiences, seek support from your peers, work collaboratively to solve problems and discuss subject material.
- Videos: There are recorded academic introductions to the subject, interviews and debates and, for some courses, audio-visual tutorials and conclusions.
- Recorded lectures: For some courses, where appropriate, the sessions from previous years' Study Weekends have been recorded and made available.

- Study skills: Expert advice on preparing for examinations and developing your digital literacy skills.
- Feedback forms.

Some of these resources are available for certain courses only, but we are expanding our provision all the time and you should check the VLE regularly for updates.

Making use of the Online Library

The Online Library contains a huge array of journal articles and other resources to help you read widely and extensively.

To access the majority of resources via the Online Library you will either need to use your University of London Student Portal login details, or you will be required to register and use an Athens login: http://tinyurl.com/ollathens

The easiest way to locate relevant content and journal articles in the Online Library is to use the **Summon** search engine.

If you are having trouble finding an article listed in a reading list, try removing any punctuation from the title, such as single quotation marks, question marks and colons.

For further advice, please see the online help pages: www.external.shl.lon.ac.uk/summon/about.php

Unless otherwise stated, all websites in this subject guide were accessed in June 2012. We cannot guarantee, however, that they will stay connected and you may need to perform an internet search to find the relevant pages.

Examination advice

Important: the information and advice given here are based on the examination structure used at the time this guide was written. Please note that subject guides may be used for several years. Because of this we strongly advise you to always check both the current *Regulations* for relevant information about the examination, and the VLE where you should be advised of any forthcoming changes. You should also carefully check the rubric/instructions on the paper you actually sit and follow those instructions.

Remember, it is important to check the VLE for:

- up-to-date information on examination and assessment arrangements for this course
- where available, past examination papers and *Examiners' commentaries* for the course which give advice on how each question might best be answered.

The examination paper consists of eight questions of which you must answer four questions. Each question carries equal marks and is divided into several parts. The style of question varies but each question aims to test the mixture of concepts, numerical techniques and application of each topic. Since topics in financial management are often interlinked, it is inevitable that some questions might examine overlapping topics.

Remember when sitting the examination to maximise the time spent on each question and although, throughout, the subject guide will give

you advice on tackling your examinations, remember that the numerical type questions on this paper take some time to read through and digest. Therefore try to remember and practise the following approach. Always read the requirement(s) of a question first before reading the body of the question. This is appropriate whether you are making your selection of questions to answer, or when you are reading the question in preparation for your answer.

In the question selection process at the start of the examination, by reading only the requirements, which are always placed at the end of a question, you only read material relevant to your choice, you do not waste time reading material you are not going to answer. Secondly, by reading the requirements first, your mind is focused on the sort of information you should be looking for in order to answer the question, therefore speeding up the analysis and saving time.

Remember, it is important to check the VLE for:

- up-to-date information on examination and assessment arrangements for this course
- where available, past examination papers and *Examiners' commentaries* for the course which give advice on how each question might best be answered.

Summary

Remember this introduction is only a complementary study tool to help you use this subject guide. Its aim is to give you a clear understanding of what is in the subject guide and how to study successfully. Systematically study the next 12 chapters along with the listed texts for your desired success.

Good luck and enjoy the subject!

Abbreviations

AEV	Annual equivalent value
AIM	Alternative investment market
APM	Arbitrage pricing model
ARD	Arnold, 2008
ARR	Accounting rate of return
BMA	Brealey, Myers and Allen
CAPM	Capital asset pricing model
CFs	Cash flows
CME	Capital market efficiency
CML	Capital market line
CPI	Consumer price index
DFs	Discount factors
DPP	Discounted payback period
DPS	Dividend per share
EMH	Efficient market hypothesis
EPS	Earnings per share
EVA	Economic value added

IPO	Initial public offer
IRR	Internal rate of return
LSE	London Stock Exchange
MM	Modigliani and Miller
MVA	Market value added
NCF	net cash flow
NPV	Net present value
NYSE	New York Stock Exchange
PE	Price earnings ratio
PI	Profitability index
PP	Payback period
ROA	Return on assets
ROC	Return on capital
ROE	Return on equity
S&P	Standard and Poor's
Std dev	Standard deviation
VLE	Virtual learning environment
WACC	Weighted average cost of capital

Chapter 1: Financial management function and environment

Essential reading

Brealey, R.A., S.C. Myers and F. Allen *Principles of corporate finance*. (New York: McGraw-Hill, 2010) tenth edition [ISBN 9780071314268] Chapters 1 and 2.

Further reading

Arnold, G. *Corporate financial management*. (Harlow: Financial Times/Prentice Hall; 2008) fourth edition [ISBN 9780273719069]. Chapter 1.

Works cited

Fisher, I. *The theory of interest*. (New York: MacMillan, 1930).

Aims

This chapter paves the foundation for you to understand what financial management is about. In particular, we will examine the roles of financial management, the environment in which businesses are operated, and agency theory. More importantly we explain the two key concepts which underpin much of the theory and practice of financial management.

Learning outcomes

By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

- outline the nature and purpose of financial management
- describe the general environment in which businesses operate
- explain the relationship between financial objectives and corporate strategies
- assess the impact of stakeholders on corporate strategies
- discuss the time value for money concept and the risk and return relationship.

Two key concepts in financial management

Before we look at what financial management is about, it is essential for us to understand two key concepts which lay the foundation of this subject. The two key concepts are:

i. Risk and return.

ii. Time value of money.

Risk and return

Financial markets seem to reward investors of riskier investments[1] with a higher return.[2] The following graph indicates this relationship.[3]

[1] Risk is often measured as a dispersion of the possible return outcomes from the expected mean. In Chapter 3 of this subject guide, we will more formally define the concept of risk in financial management and discuss the different methods to quantify risk.

[2] Return refers to the financial reward gained as a result of making an investment. It is often defined as the percentage of value gain plus period cash flow received to the initial investment value.

[3] The graph has been rescaled in log to fit the page. You should note the vast differences of the cash returns from each investment type.

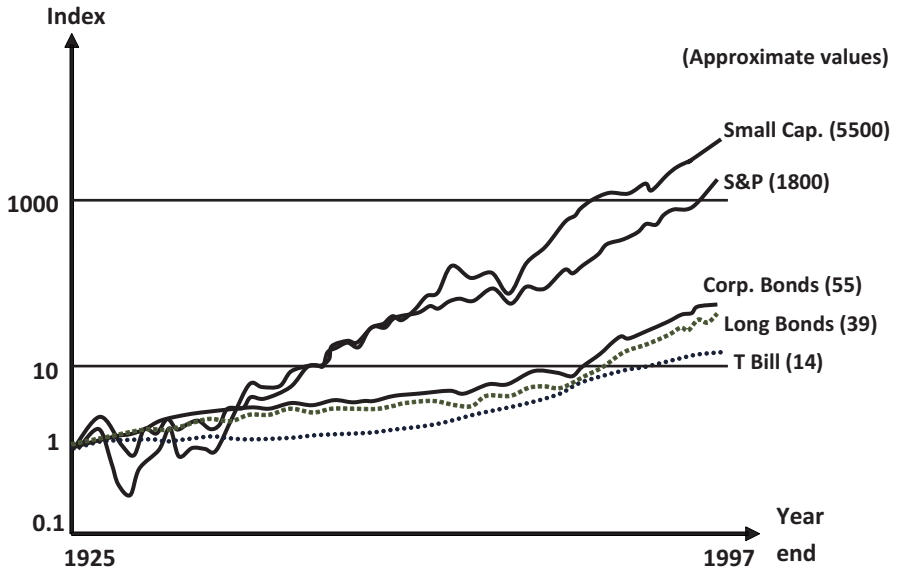

Figure 1.1: The cash return from five different investments.

Source: BMA.

Suppose we invested $1 in 1925 in each of the following five portfolios:

i. the largest quoted companies in the US, Standard & Poor's (S&P)

ii. the smallest quoted companies measured by market capitalisation in the US, Small Capitalisation (Small Cap)

iii. corporate bonds

iv. long-term US government bonds, Long Bonds

v. short-term US government bonds, T Bill.

These portfolios have different levels of perceived risk. Arguably, smaller companies have higher varying returns than larger companies. Bonds, on the other hand, are a safer investment to investors. Over time, these portfolios generate cash returns which seem to follow the same order as their respective perceived risk. This leads us to one of the axioms in financial management:

The higher the risk, the higher the expected return.

Companies and investors should therefore only consider undertaking a riskier investment provided that they are suitably and sufficiently compensated by a higher return.

Activity 1.1

What are the main reasons for smaller companies having higher perceived risk? What are the specific risks we are referring to?

See VLE for discussion.

Time value of money[4]

Money (i.e. cash) has different values over time. Holders of money can either spend a sum of money now or delay their consumption by investing the money in different investment opportunities until it is required.

Suppose an investor can deposit a sum of money in a bank and earn an annual interest of 5%. The value of money to this investor would then be 5% per annum. If the same investor can invest the same sum of money in a financial asset which gives a return of 10% annually, then the value of money to this investor would be 10% per annum. The future return from

[4] BMA, Chapter 2 deals with the concept of time value for money and covers in detail how to calculate present and future values.

the money invested now is based on the duration of time, the risk of the investment and inflation.

For example, $100 invested today will earn 10% per annum of return (i.e. $110 in one year's time and $121 in two years' time). An investor who assumes a 10% return will be indifferent between receiving $100 today and $110 in one year's time as the two cash flows have identical value to the investor. In the time value of money terminology, the **present value** of $110 received in one year's time is exactly $100. Similarly, the present value of $121 received in two years' time is exactly $100, too.

This concept can be applied to convert future cash flows into their present values. Denote the present value of a cash flow as *PV* and future (*t*-period) value of a cash flow as FV_t. The general relationship between the present and future value is:

$FV_t = PV(1+r)^t$ where *r* is the time value of money measured as a percentage

Re-arranging the above equation, we have:

$$PV = \frac{FV_t}{(1+r)^t} = FV_t \times \frac{1}{(1+r)^t}$$

where $\frac{1}{(1+r)^t}$ is the *t*-period discount factor

The nature and purpose of financial management

Having discussed the two key concepts in financial management, we can now turn our attention to the function of financial management. In general, there are three main tasks that financial managers need to undertake:

i. Investing decisions – this is how financial managers select the 'right' investments. This can be examined in two stages. First we look at how financial managers invest in and manage short-term working capital (this is covered in Chapter 11 of this subject guide) and then we examine how financial managers may appraise long-term investment projects.

ii. Financing decisions – this involves the choice of particular sources of funds which provide cash for investments. The key issues that financial managers should address are how:

   - these sources of funds can be raised (covered in Chapter 5)
   - the value of the business may be affected through the combination of different sources of funds (covered in Chapter 6)
   - the sources of funds may affect the relationship between different stakeholders (covered in Chapter 6).

iii. Dividend policy – this concerns the return to shareholders (covered in Chapter 7).

So in theory and in practice, how are these decisions being considered by financial managers?

Link between investing, financing and dividend decisions

In a perfect and complete capital market where there are no transaction costs and information is widely available to everyone, it is argued that a firm's investing, financing and dividend decisions are not interlinked. This is known as Fisher's separation theorem (Fisher, 1930). This is illustrated in the following diagram.

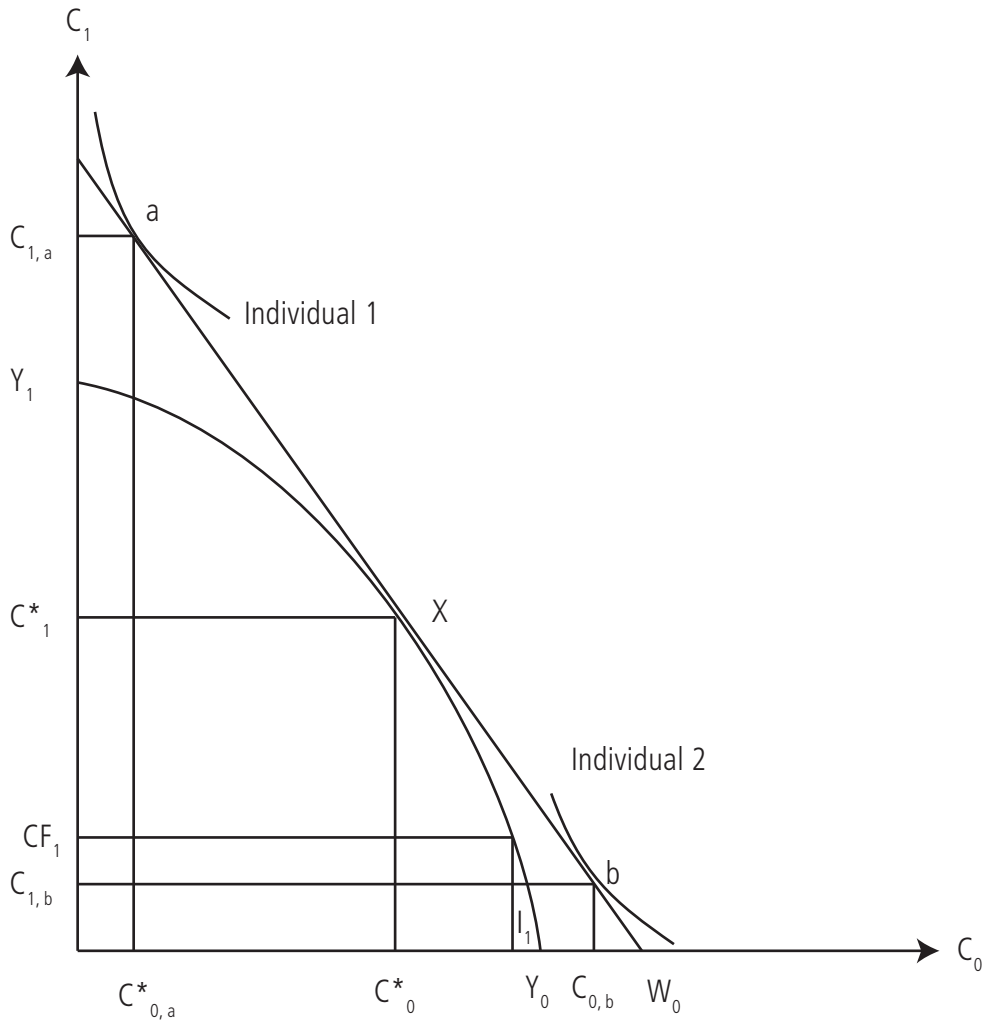

Figure 1.2: Fisher's Separation Theorem

Suppose a firm is operating in a two-period environment (period 0 – now and period 1 – in one year's time) with an initial cash flow of Y_0. It has the opportunity to invest in two types of investments. The first type of project relates to investments which require an initial investment outlay (I_i) and deliver CF_i in the next period for each investment (i). For example, investing I_i in period 0 will produce CF_i in period 1. Hereafter these types of projects are referred to as production investment projects. The second type of investment is essentially financial, which allows the firm to borrow and lend an unlimited amount at an interest rate of r. In this case, if a firm borrows (or lends) W_0 in period 0, it will pay back with interest (or receive with interest) $W_1 = W_0(1+r)$.

Investing decision

What should the firm do in terms of its investments? A firm will logically rank and invest in investment projects in descending order of their profitability (R_i for each i). A production opportunity frontier can be obtained (such as the curve Y_0Y_1). A firm will invest up to the point where the marginal investment i^* yields a return that equals the return from the capital market (i.e. interest rate r). The total investment outlays - the amount represented by C_0Y_0 - is the sum I_i for all i (i = 1 to i^*). Once the investment plan is fixed, the firm will have C^*_0 in period 0 remaining and a cash return of C^*_1 in period 1.

Dividend policy

In this setting, how much should the firm give out as dividend to its shareholders in each period? The answer is simple. It should give out C^*_0 and C^*_1 in period 0 and 1 respectively. However, would shareholders be satisfied with these amounts in each period? Suppose we have two individual shareholders 1 and 2. Each of them has their unique utility function of consumption in each period. This can be represented by the indifference curves in Figure 1.2 above. Individual 1 prefers to consume less in period 0 and more in period 1 (the combination at 'a'). Given the current firm's dividend policy, how would he be satisfied? There are two ways to achieve it:

i. The firm will pay $C_{0,a}$ and invest any excess cash flow (i.e. $C^*_0 - C_{0,a}$) at r in period 0 and give out $C^*_1 + (C^*_0 - C_{0,a})(1 + r)$. Mathematically, it can be proved that it is equal to $C_{1,a}$. Therefore the firm will pay the exact dividend in each period to individual 1 as he prefers.

ii. Alternatively, the firm pays C^*_0 to individual 1 and he can invest any excess cash flow after his consumption in period 0 in the financial investment earning a return of r and receive the same combined cash flow of $C_{1,a}$ in period 1.

This reasoning applies to any individual shareholders with any unique utility functions. Take Individual 2 as an example. Her consumption pattern does not match the firm's dividend payout. Similarly there are two ways we can satisfy her consumption pattern:

i. The firm will borrow $C_{0,b} - C^*_0$ at r in period 0 and pay out $C_{0,b}$ to Individual 2. In period 1, the firm will pay out $C^*_1 - (C_{0,b} - C^*_0)(1 + r)$. Mathematically, it can be proved that it is equal to $C_{1,b}$. Therefore the firm will pay the exact dividend in each period to Individual 2.

ii. Alternatively, the firm pays C^*_0 to Individual 2 and she borrows any shortfall to make up to her consumption $C_{0,b}$ in period 0. In period 1, she will receive C^*_1 less the loan and interest she takes out in period 0. This will leave her with a net amount exactly equal to $C_{1,a}$.

The above argument indicates that financial managers do not need to consider shareholders' consumption patterns when fixing the investment plan or the dividend policy. The easiest way is to maximise the firm's cash flows and distribute the spare cash flows as dividends. Shareholders will use the capital markets to facilitate their consumption patterns accordingly.

Financing decision

In the beginning, we assume that the firm has an initial cash flow of Y_0 and requires a total investment outlay of $C_0 Y_0$. If any part of Y_0 is not contributed by shareholders, the firm's dividend in period 1 will be reduced by the funds raised from borrowing (at a cost of r) and the interest. However, shareholders can offset this shortfall of dividend in period 1 by investing the fund not contributed in the firm to the capital market and earn a return exactly equal to r.

The above argument illustrates the Fisher separation in which investing, financing and dividend decisions are all unrelated. However, if the capital market is imperfect in such a way that external funding is restricted, the Fisher separation might not apply. The following scenarios highlight the practical considerations that financial managers would need to take.

Investment A company would like to undertake a large number of profitable investment projects.	**Financing** It will need to raise funds in order to take up these projects.	**Dividends** If the company fails to raise sufficient funds from outside the company, it would need to cut dividends in order to increase internal funding.
Dividends A company wants to pay a large dividend to shareholders	**Financing** A lower level of available internal cash flows might force the company to seek extra funds via external financing.	**Investment** If external financing is restricted through partially financing the dividend, the company might need to postpone some of the investment projects.
Financing A company has been using a higher level of external funding.	**Investment** Due to the high cost of financing, the number of attractive investment projects might be reduced.	**Dividends** The company's ability to pay dividends in the future may be adversely affected.

Activity 1.2

i. Why would a firm invest up to the point where the return of the marginal investment equals the return from the capital market?

ii. What would happen to the Fisher's separation theorem if the borrowing rate differs from the lending rate?

See VLE for solutions.

Corporate objectives

BMA, Chapter 1, pp.37–40 discuss the goals of corporation. The general assumption in financial management is that corporate managers will try their best to maximise the value of the shareholders' investment in the corporation (i.e. shareholders' wealth maximisation (SHWM)). Maximisation of a company's ordinary share price is often used as a surrogate objective to that of maximisation of shareholder wealth. In order to achieve this objective, it is argued that corporate managers will maximise the value of all investments undertaken by the firm. This can be illustrated in the following diagram:

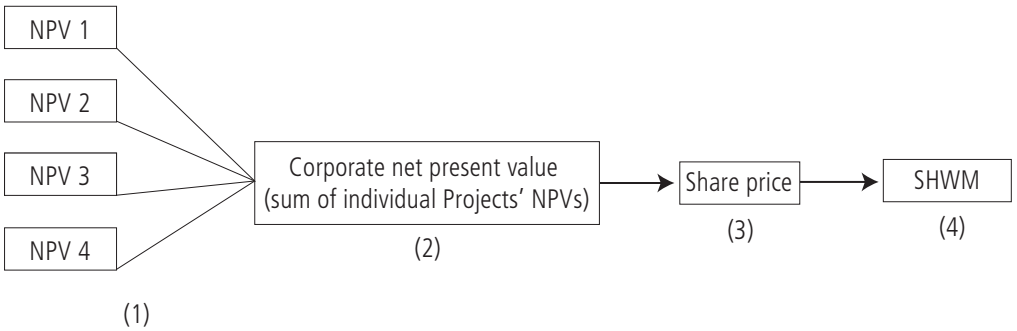

Figure 1.3: Shareholders' wealth maximisation

Source: BMA.

However, in practice, corporate objectives vary. For example, HP, a US-based computer corporation, has the following objectives listed on its website:[5]

[5] *(http://welcome.hp.com/country/uk/en/companyinfo/corpobj.html)*

- customer loyalty
- profit
- growth
- market leadership
- leadership capability
- employee commitment
- global citizenship.

While profit maximisation, social responsibility and growth represent important supporting objectives, the overriding objective of a company must be that of shareholders' wealth maximisation. The financial wealth of a shareholder can be affected by a company's financial manager's action. Arguably, when good investment, financing and dividend decisions are made, a company's market value will increase. The rest of this subject guide will explore how financial managers' decisions can increase a firm's value.

Activity 1.3

Although shareholders' wealth maximisation seems to be the overriding objective, corporate managers still face a number of constraints to implement multiple objectives simultaneously.

Identify the types of constraint that corporate managers face when assessing long-term financial plans.

See VLE for discussion.

The agency problem

The agency problem occurs when financial managers make decisions which are not consistent with the objectives of the company's stakeholders. It arises because:

1. There is a separation of ownership and control: agents (financial managers) are given the power to manage and control the company by the principals (stakeholders: shareholders, creditors and customers).
2. The goals of agents are different from those of the principals.[6]
3. Principals do not get full information about their company from the agent or the market (asymmetric information).

[6] For example, agents may want to increase the size of the company (empire building), strengthen their managerial power, secure their jobs, improve their remuneration and pursue other personal objectives. These objectives may not necessarily be enhancing the value of the company.

Activity 1.4

What are the signs of an agency problem? What possible actions can be taken to mitigate such a problem?

See VLE for discussion.

A reminder of your learning outcomes

Having completed this chapter, as well as the Essential reading and activities, you should be able to:

- outline the nature and purpose of financial management
- describe the general environment in which businesses operate
- explain the relationship between financial objectives and corporate strategies
- assess the impact of stakeholders on corporate strategies

- discuss the time value for money concept and the risk and return relationship.

Practice questions

1. Compute the future value of $1,000 compounded annually for:

   a. 10 years at 5%

   b. 20 years at 5%

   How would your answer to the above question be different if interest is paid semi-annually?

2. Compare each of the following examples to a receipt of $100,000 today:

   a. Receive $125,000 in two year's time.

   b. Receive $55,000 in one year's time and $65,000 in two year's time

   c. Receive $31,555.7 for the next 4 years, receivable at the end of each year.

   d. Receive $10,000 for each year for an infinite period.

   Assume the interest rate is 10% per year for the foreseeable future.

Sample examination questions

1. 'We need to maximise our profit in order for us to maximise the shareholders' wealth' – Executive at OverHill Plc.

   Critically comment on the statement above.

2. Explain, with the aid of a diagram, how a firm's dividend policy is independent from its investment policy in a perfect and complete world.

3. Identify five different stakeholder groups of a public company and discuss their financial and other objectives.

Chapter 2: Investment appraisals

Essential reading

Brealey, R.A., S.C. Myers and F. Allen *Principles of corporate finance*. (New York: McGraw-Hill, 2010) tenth edition [ISBN 9780071314268] Chapters 5 and 6.

Further reading

Arnold, G. *Corporate financial management*. (Harlow: Financial Times/Prentice Hall, 2008) fourth edition [ISBN 9780273719069] Chapters 2–6.

Works cited

Graham, J.R. and C.R. Harvey 'The theory and practice of corporate finance: evidence from the field', *Journal of Financial Economics*, 60, 2001, pp.187–243.

Aims

This chapter focuses on the techniques commonly used for investment appraisals in practice. In particular, we concentrate on the pros and cons of the following techniques:

- Accounting rate of return (ARR)
- Payback period (PP)
- Discounted payback period (DPB)
- Internal rate of return (IRR)
- Net present value (NPV).

Learning outcomes

By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

- describe the commonly used investment appraisal techniques
- apply the discounted cash flow technique in complex scenarios
- evaluate the investment decision process.

Overview

As mentioned in Chapter 1, financial managers make decisions about which investment they should invest in to maximise their shareholders' value. In order to do so, they need to understand how to measure the value of investments they undertake and how these investments help to improve the value of the firm. First, we will examine the basic techniques and evaluate their pros and cons in investment appraisals. We will then compare the relative merits of using NPV over IRR. Thirdly, we consider some of the scenarios when NPV can be applied to deal with the selection of investments. Finally, we discuss the problems relating to the application of these investment appraisal techniques.

Basic investment appraisal techniques

BMA, Chapter 5 reviews the appraisal techniques and explains them at great length. You should read the relevant sections of the chapter before you carry on with the rest of the material covered here.

Here we summarise these commonly used techniques.

Accounting rate of return (ARR)

The method is also known as **return on capital employed (ROCE)** or return on investment (ROI). It relates accounting profit to the capital invested. One widely used definition is:

$$\text{ARR} = \frac{\text{Average annual profit}}{\text{Average investment outlays}} \times 100\ \%$$

Average investment takes into consideration any scrap value. It can be expressed as follows:

$$\text{Average Investment} = \frac{\text{Investment} - \text{Scrap value}}{2}$$

It measures the average net investment outlay of the project.[1] Accounting profit is defined as before-tax operating cash flows after adjustment for depreciation. The decision rule is to accept investments with ARR higher than a predetermined target rate of return.

[1] Some textbooks prefer to calculate ARR by referring to the average level of investment. Consequently, the average investment will be defined as (initial investment + scrap value)/2.

Payback period (PP)

Payback period measures the shortest time to recover the initial investment outlay from the cash flows generated from the investment. A company will accept an investment if the PP is less than or equal to a target period.

Discounted payback period (DPP)

This is similar to PP except that the cash flows from the investment are first discounted to time 0 and the shortest time to recover the initial investment outlay will then be measured.

Internal rate of return (IRR)

The internal rate of return on an investment or project is the annualised effective compounded return rate or discount rate that makes the net present value (NPV) of all cash flows (both positive and negative) generated from a particular investment equal to zero. The decision rule is to accept a project or investment if its IRR is higher than the cost of capital.

Net present value (NPV)

NPV combines the present values of all future cash flows and compares the total to the initial investment. If the NPV of a project is positive, it indicates that it earns a positive return over the cost of capital and will therefore increase the shareholders' value. A firm should invest in all positive NPV projects, so the market value of the firm will increase by the total of the NPVs, once they are announced to the market.

To illustrate how these techniques are applied in investment appraisal, let's look at the following example.

Example 2.1

Suppose we have two mutually exclusive projects, A and B. Each project requires an initial investment in a machine, payable at the beginning of year 0. There is no scrap value for these machines at the end of the project. Suppose the cost of capital (discount rate) is 20% per annum. The following before-tax operating cash flows are also known:

Before-tax operating cash flows ($)	Year				
Project	**0**	**1**	**2**	**3**	**4**
A	(25,000)	5,000	10,000	15,000	20,000
B	(2,500)	2,000	1,500	250	

Accounting rate of return

Suppose the profit before depreciation for each year is identical to the annual cash flow. The ARR can be determined as follows:

Project	Initial investment	Average investment	Total profit after depreciation	Average profit	ARR
A	25,000	12,500	25,000	6,250	50%
B	2,500	1,250	1,250	417	33%

Payback period

We can look at the cumulative cash flow at the end of each year to determine the PP.

Cumulative cash flows						
Project	**0**	**1**	**2**	**3**	**4**	**PP**
A	(25,000)	(20,000)	(10,000)	5,000	25,000	2.67 years
B	(2,500)	(500)	1,000	1,250		1.33 years

Discounted payback period

	Year				
Project A	**0**	**1**	**2**	**3**	**4**
Cash flows ($)	(25,000)	5,000	10,000	15,000	20,000
Discount factor (DF) (20%)	1	0.833	0.694	0.578	0.482
Present value	(25,000)	4,165	6,940	8,670	9,640
Cumulative cash flows	(25,000)	(20,835)	(13,895)	(5,225)	4,415
	Year				
Project B	**0**	**1**	**2**	**3**	**4**
Cash flows ($)	(2,500)	2,000	1,500	250	
Discount factor (DF) (20%)	1	0.833	0.694	0.578	0.482
Present value	(2,500)	1,666	1,041	144.5	
Cumulative cash flows	(2,500)	(834)	207		

For Project A, the payback period occurs in Year 4. If we assume that cash flows arrive evenly throughout the year, we can determine the approximated payback period at 5,225/9,640 = 0.54 year (i.e. PP at 3.54 years). Similarly, for Project B, the PP occurs in 1.8 years.

Net present value

The NPV can be determined as:

			Year		
Project A	**0**	**1**	**2**	**3**	**4**
Cash flows ($)	(25,000)	5,000	10,000	15,000	20,000
Discount factor (DF) (20%)	1	0.833	0.694	0.578	0.482
Present value	(25,000)	4,165	6,940	8,670	9,640
NPV	4,415				
			Year		
Project B	**0**	**1**	**2**	**3**	**4**
Cash flows ($)	(2,500)	2,000	1,500	250	
Discount factor (DF) (20%)	1	0.833	0.694	0.578	0.482
Present value	(2,500)	1,666	1,041	144.5	
NPV	351.5				

Internal rate of return

To find the IRRs of these two projects, we can use the extrapolation method. First, we recalculate the NPV of each of the two projects with a higher discount rate. For example, we choose 30% and 35% as the discount rate for Project A and B respectively. This gives, in both cases, negative NPVs.

			Year		
Project A	**0**	**1**	**2**	**3**	**4**
Cash flows ($)	(25,000)	5,000	10,000	15,000	20,000
Discount factor (DF) (30%)	1	0.769	0.592	0.455	0.35
Present value	(25,000)	3,845	5,920	6,825	7,000
NPV	(1,410)				
			Year		
Project B	**0**	**1**	**2**	**3**	
Cash flows ($)	(2,500)	2,000	1,500	250	
Discount factor (DF) (35%)	1	0.741	0.549	0.407	
Present value	(2,500)	1,482	824	102	
NPV	(93)				

We then substitute the relevant figures into the following equation:

$$IRR = R^{+} + \frac{NPV_{R^{+}}}{NPV_{R^{+}} - NPV_{R^{-}}}\left(R^{-} - R^{+}\right)$$

R^{+} is the discount rate which gives a positive NPV, NPV_{R+}

R^{-} is the discount rate which gives a negative NPV, NPV_{R-}

Consequently, the IRRs for Project A and B are 27.6% and 31.9% respectively.

Activity 2.1

Attempt Question 1, BMA Chapter 5.

See VLE for solution.

Pros and cons of investment appraisal techniques

Example 2.1 highlights the potential problems of using some of these techniques in investment appraisals. Recall the results for Projects A and B respectively:

Projects	NPV	IRR	PP	ARR
A	4,415*	27.6%	2.67 years	50%*
B	351.5	31.9%*	1.33 years*	33%

** Indicates the project that will be chosen under the specific appraisal method.*

Suppose the main objective is to maximise shareholders' value. Financial managers would prefer Project A as it provides a higher NPV, and hence it gives the greatest increase to the shareholders' value. However, if we choose projects based on a higher value of IRR or PP, Project B will be selected. But this project clearly does not produce the greatest value to the company. So why are these techniques still being used in practice?

ARR

Advantages:

- It gives a value in percentage terms which is a familiar measure of return.
- It is relatively easy to calculate compared to NPV or IRR.
- It considers the cash flows (but only after adjustment for depreciation in profit) arising from the lifetime of the project (unlike PP).
- It can be used in selecting mutually exclusive projects.

Disadvantages:

- It is very much based on the accounting profits and hence technically it does not deal with the actual cash flows arising from the project.
- It ignores the timing of the cash flows and hence it does not take into consideration the time value of money.
- It is expressed in percentage terms and therefore it does not measure the absolute value of the project. It does not indicate how much wealth the project creates.

PP

Advantages:

- It is computationally straightforward.
- It considers the actual cash flows, not profits, arising from a project.

Disadvantages:

- It ignores cash flows beyond the PP and hence it does not provide a full picture of a project.
- It does not consider the time value of money (even though the discounted payback period takes care of that).
- The target payback period is somehow arbitrary.

IRR

Advantages:

- It uses all relevant cash flows, not accounting profits, arising from a project.
- It takes into account the time value of money.

- The difference between the IRR and the cost of capital can be seen as a margin of safety.

Disadvantages:

The main limitations of using IRR in investment appraisals are that it may not give the correct decision in the following scenarios:

- when comparing mutually excusive projects
- when projects have non-conventional cash flows
- when the cost of capital varies over time
- It discounts all flows at the IRR rate not the cost of capital rate.

Mutually exclusive projects

Referring to Example 2.1, Project B's IRR is higher than that of Project A. One would rank Project B as more 'desirable' than Project A. However, if we consider the NPV of these projects, there is no doubt that Project A is, by far, more valuable than Project B.

Non-conventional cash flows

A typical investment project has an initial cash outflow followed by positive cash flows in subsequent years. However, in some cases, a project (such as oil drilling or mining) may have negative cash flows during its lifetime. Mathematically, each time the cash flow stream of a project changes sign, there is a possibility that multiple IRRs might arise.

Example 2.2

Suppose a project requires $100 as an initial investment. Its Year 1 and Year 2 cash flows are $260 and –$165 respectively. Based on this project's cashflows, it produces two possible IRRs (10% or 50%):

		DF	PV	DF	PV
Year	Cash flows	50%		10%	
0	–100	1	–100	1	–100
1	260	0.667	173	0.909	236
2	–165	0.445	–73	0.826	–136
Net Present Value			0		0

Suppose the cost of capital for this project is 20%. According to the IRR rule, the project should be accepted (as the cost of capital is less than the higher IRR of 50%). However, it should also be rejected as the cost of capital is higher than the lower IRR of 10%. So for a project with non-conventional cash flows, the IRR decision is sensitive to the cost of capital. Therefore it is argued that IRR does not give an unambiguous decision when dealing with non-conventional projects.

To further illustrate this problem, let's look at the NPV profile of the project. This depicts the relationship of the NPV of the project and its discount rate. In the above example, we know that the NPV of the project is zero at both 10% and 50%.

Suppose the cost of capital is 5%, 25% or 70%. The NPV of the project will become –$2, $2 and –$4 respectively. The following diagram shows the NPV profile of the project. We can see that, due to the non-conventional cash flow pattern, the project's NPV varies at different discount rates. It only provides a positive NPV if the discount rate for the project's cash flows is between 10% and 50%.

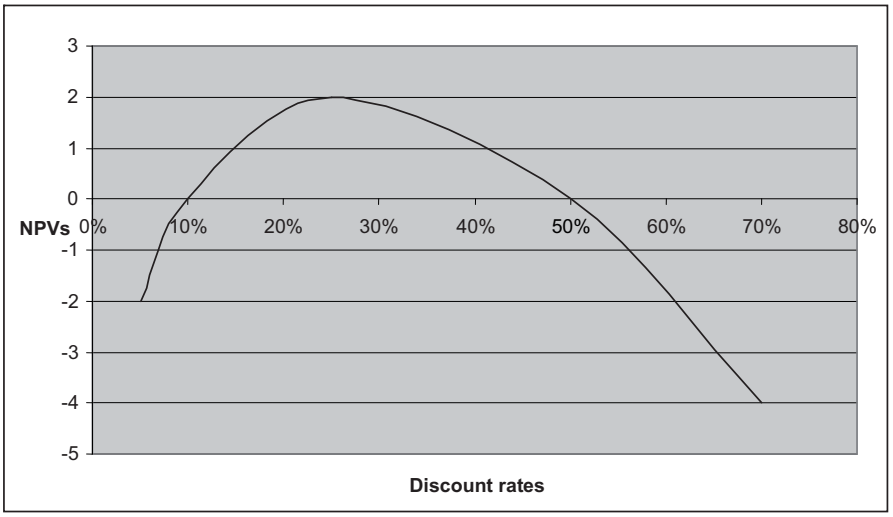

Figure 2.1: NPV profile

However, if the project we have been examining has the 'reversed' cash flow pattern (i.e. receiving \$100 and \$165 in year 0 and year 2 while paying \$260 in year 1), we would only accept it if the cost of capital is either lower than 10% or higher than 50%. Why? This project with the reversed cash flow pattern has the same IRRs (10% and 50%) as the original project. You can verify this result by discounting the cash flows at 10% and 50% separately. However, the NPV profile of this project will be as below.

Time-varying cost of capital

If the cost of capital changes over time, NPV can easily accommodate this. Suppose the cost of capital is r_t for the t^{th} year. The NPV of a project with different cost of capital over its lifetime can be given in the following equation:

$$NPV = -I_0 + \frac{C_1}{(1+r_1)} + \frac{C_2}{(1+r_1)(1+r_2)} + \frac{C_3}{(1+r_1)(1+r_2)(1+r_3)} + \ldots$$

NPV assumes that cash flows can be reinvested at the cost of capital whereas IRR assumes that cash flows can be reinvested at the IRR which is not a realistic assumption in the real world.

The superiority of NPV:

- It takes into consideration all cash flows and time value of money.
- It can be applied to deal with mutually exclusive projects.
- It can deal with non-conventional cash flows.
- It has realistic assumptions about how the capital markets work in real life.

Activity 2.2

Attempt Question 5, BMA Chapter 5.

See VLE for solution.

Advanced investment appraisals

In this section, we look at some of the applications of the discounted cash flow technique in investment appraisals. In particular, we focus on the following scenarios:

- capital rationing
- inflation and price changes
- taxation
- replacement decision

- sensitivity analysis.

BMA, Chapter 5, pp.143–47 deals with capital rationing and Chapter 6 deals with the remaining advanced topics. Before you proceed with the following section, it would be advisable to skim through those sections in the textbook.

Capital rationing

A company may have insufficient funds to undertake all positive NPV projects. Due to the shortage of funds, this restriction is more commonly known as capital rationing. There are two types of capital rationing.

Hard capital rationing

This is where the shortage of funds is imposed by external factors. This might happen in three different ways:

1. Capital markets are depressed.
2. Investors are too risk adverse.
3. Transaction costs are too high.

Soft capital rationing

This may arise when financial managers impose internal restrictions on:

- issuing equity to avoid dilution of original shareholders' value
- issuing debt to avoid fixed interest obligation and transaction cost
- investing activities in order to maintain a constant growth.

In any case, ranking projects by absolute NPV in these situations may not necessarily give the optimal strategy. Some combinations of smaller projects may give a higher NPV.

For each type of capital rationing we can further sub-divide it into two categories.

Single period capital rationing

If the shortage of funds is only restricted in the first year, the ranking of projects can be done by using the profitability index. Profitability index is defined as the present value of the future cash flows generated by a project divided by its initial investment. It is also called the Present Value Index (PVI) by some authors.

$$\text{Profitability index, PI} = \frac{\text{Present value of future cash flows}}{\text{Initial investment}}$$

Example 2.3

Lion plc has the following projects:

Projects	Initial Investment ($)	NPV ($)
A	1,000,000	100,000
B	1,500,000	250,000
C	750,000	50,000
D	500,000	60,000

The company has only $2,500,000 available at year 0. There is no other investment opportunity for the firm with any spare cash which is not invested in the above four projects.

What would be the best way to allocate the $2,500,000 funding among these four projects?

To answer this question, we first convert the NPV into PV (Initial investment + NPV) for each project. We then calculate the PI using the above formula.

Projects	Initial Investment ($)	NPV ($)	PV ($)	PI	Ranking
A	1,000,000	100,000	1,100,000	1.10	3
B	1,500,000	250,000	1,750,000	1.17	1
C	750,000	50,000	800,000	1.07	4
D	500,000	60,000	560,000	1.12	2

In this case, the ranking of the project's profitability is simple and straightforward. The PI suggests that for every $1 invested in Project B, it produces a present value of $1.17. When this is compared to Project A's PI, it is obvious that for any $1 available, it is more profitable to invest in Project B than in Project A.

When projects are infinitely divisible

The optimal plan is to invest all the available cash in the projects according to the ranking of PI. In this case, we will invest in the whole of Project B and Project D (with a combined total initial investment of $2,000,000) and in half of Project A with the remaining $500,000. The maximum NPV of this investment plan is:

$$\begin{aligned}\text{The optimal NPV} &= \$250{,}000 + \$60{,}000 + \frac{1}{2} \times \$100{,}000 \\ &= \$360{,}000\end{aligned}$$

When projects are not infinitely divisible

When projects are not infinitely divisible, the above investment plan might not necessarily be optimal as the spare cash of $500,000 would no longer be investable in only half of Project A. The optimal investment plan would therefore involve a strategy which gives the highest PI to the investment plan. Note that any unused cash in the investment plan, by definition, has a PI = 1 (the present value of the unused cash is the same as the amount of the unused cash itself). We can define the weighted average of the investment plan as:

$$WAPI = \sum_{i=1}^{N} \omega_i PI_i + \omega_j$$

where ω_i is the percentage of project i's initial investment to the total cash available, PI_i is the profitability index of project i, and ω_j is the percentage of unused cash to the total cash available.

Weight	Plan					
Project	A+B	A+C	A+C+D	B+C	B+D	C+D
A	0.4	0.4	0.4	0	0	0
B	0.6	0	0	0.6	0.6	0
C	0	0.3	0.3	0.3	0	0.3
D	0	0	0.2	0	0.2	0.2
Unused cash	0	0.3	0.1	0.1	0.2	0.5
WAPI	1.14	1.06	1.09	1.12	1.13	1.05

The highest combination is to undertake both Projects A and B. This gives a weighted average PI of 1.14. It means for every $1 we invest, we will receive $1.14 of future cash measured at today's value.

Multiple periods capital rationing

When a firm is facing multiple periods of capital rationing, it would not be easy to resolve the optimal investment plan by using the profitability index. In this case, linear programming technique might be useful.

Activity 2.3

Attempt Question 7, BMA Chapter 5.

See VLE for solution.

Changing prices and inflation

The accuracy of NPV depends on the accuracy of the cash flow estimates. In practice, prices change for the following reasons:

- inflationary effect
- demand and supply
- technological changes
- manufacturing learning effect
- stamp duties, value-added tax and other transaction costs.

The easiest way to deal with these external effects is to incorporate the specific changes in the NPV calculation, i.e. the forecast for each period's flows will be based on each flow item adjusted by its specific inflation to give the project actual net flow for each period.

Example 2.4

Suppose Leopard plc has a project that produces 10,000 units of a digital diary per year for the next four years. Each unit sells for $200. The unit production cost is $110. The production requires a brand new machine at year 0. It costs $2,000,000 with a scrap value of $20,000 at the end of year 4. The NPV of this project (assuming no inflation) is determined as follows:

	Year				
	0	**1**	**2**	**3**	**4**
Machine	(2,000,000)				20,000
Revenue		2,000,000	2,000,000	2,000,000	2,000,000
Production costs		(1,100,000)	(1,100,000)	(1,100,000)	(1,100,000)
NCF before tax	(2,000,000)	900,000	900,000	900,000	920,000
DF	1	0.909	0.826	0.751	0.683
PV	(2,000,000)	818,100	743,400	675,900	628,360
NPV	865,760				

Example 2.5

Suppose the production cost for each unit will rise by 10% per year from year 2 onward. The revised NPV of this project can be determined by incorporating the price changes to the production costs in Example 2.4.

	Year				
	0	1	2	3	4
Machine	(2,000,000)				20,000
Revenue		2,000,000	2,000,000	2,000,000	2,000,000
Production costs		(1,100,000)	(1,210,000)	(1,331,000)	(1,464,100)
NCF before tax	(2,000,000)	900,000	790,000	669,000	555,900
DF (10%)	1	0.909	0.826	0.751	0.683
Present value (PV)	(2,000,000)	818,100	652,540	502,419	379,680
Net present value (NPV)	352,739				

The effect of this price change to the manufacturing costs reduces the NPV from $865,760 to $352,739. If financial managers fail to recognise and take this price change into consideration, it is very likely that the project's NPV will be grossly misstated and an incorrect decision might be reached.

Taxation

When a firm is making a profitable investment, it is likely that it will be liable for corporate tax. When evaluating a project, the tax effect must be considered. There are two issues relating to the after-tax NPV of a project:

The amount of tax payable

Different countries have different tax rules. Generally, corporate tax is payable as a percentage of the taxable profit determined by the tax authority. In principle, most items that are charged to the Statement of Comprehensive Income (more commonly known as a Profit and Loss Account in the UK) are tax deductible. However, in some countries, the accounting depreciation for capital expenditure is not a recognised expense for tax purposes. If such a depreciation charge is not allowed, the tax authority might give an allowance for capital expenditure. For the purpose of this course, we assume that the taxable profit before capital allowance is identical to the annual net cash flow. Capital allowance is then determined as a percentage of the written down value of the capital expenditure (i.e. initial investment).

Example 2.6

Suppose Leopard plc in Example 2.4 pays corporate tax at 45% on taxable profits after capital allowances. We are told that the annual capital allowance is determined at 25% of the written down value at the beginning of each year. Any unrelieved written down value in the final year of the project is given out as capital allowance in full in that year. The following table shows the calculations of the annual capital allowance and tax payable.

	Year				
	0	1	2	3	4
Taxable profit before capital allowances		900,000	790,000	669,000	555,900
Written down values (WDVs)	2,000,000	1,500,000	1,125,000	843,750	
Capital allowances (CAs)		(500,000)	(375,000)	(281,250)	(843,750)
Taxable profit after capital allowances		400,000	415,000	387,750	(287,850)
Tax (45%)		(180,000)	(186,750)	(174,488)	129,533

The first year's capital allowance is calculated as 25% of the written down value of the initial investment (i.e. 25% × $2,000,000 = $500,000). This is then deducted from the taxable profit before capital allowances (i.e. the net cash flow of year 1) to arrive at the taxable profit after capital allowances (i.e. $900,000 – $500,000 = $400,000). The tax charge for the first year is calculated as 45% of $400,000 (i.e. $180,000).

For years 2 and 3, the same approach for the calculation of capital allowances and tax charges applies. However, at the beginning of year 4, the unrelieved written down value of the initial investment ($843,750) will be treated as the capital allowance for that year. This gives rise to a negative figure for the taxable profit after capital allowances. If Leopard plc has sufficient profits from its other operations, it can use this 'tax relief' to reduce the tax charge for the other parts of its operations, saving the company from paying taxes of $129,533 (45% of $287,850). Given that this tax saving is generated as a result of this project, it should therefore be considered as a relevant cash flow for this project's NPV.

The timing for tax payable

In Example 2.6, we determined how much tax Leopard had to pay. However, we did not discuss the second issue of **when** tax should be paid. Why is it important to determine the timing of tax payable? Recall the concept of time value of money. Cash flows, whether positive or negative, arising at different time periods would have an effect on a project's NPV. Regarding tax payables, the further away from today we settle the tax liabilities, the less impact the tax will have on the project's NPV. To see this effect, let us consider the following two cases:

Case 1: Tax payable in the same year as the profit to which it is related

	Year				
	0	1	2	3	4
Machine	(2,000,000)				20,000
Revenue		2,000,000	2,000,000	2,000,000	2,000,000
Production costs		(1,100,000)	(1,210,000)	(1,331,000)	(1,464,100)
NCF before tax	(2,000,000)	900,000	790,000	669,000	555,900
Tax		(180,000)	(186,750)	(174,488)	129,533
NCF after tax	(2,000,000)	720,000	603,250	494,513	685,433
DF	1	0.909	0.826	0.751	0.683
PV	(2,000,000)	654,480	498,285	371,379	468,150
NPV	(7,706)				

In this case, taxes are paid in the same year as the profits to which they are related. The amount of taxes paid reduces the net cash flow of the project. Note that the tax saving in year 4 is included as a positive cash flow. The after-tax NPV of this project (after discounting) is now –$7,706, suggesting that it should not be accepted. We can clearly see in this case that the tax effect on a project's acceptability cannot be ignored as it turns the positive NPV into negative.

Case 2: Tax payable one year in arrears

	Year					
	0	**1**	**2**	**3**	**4**	**5**
Machine	(2,000,000)				20,000	
Revenue		2,000,000	2,000,000	2,000,000	2,000,000	
Production costs		(1,100,000)	(1,210,000)	(1,331,000)	(1,464,100)	
NCF before tax	(2,000,000)	900,000	790,000	669,000	555,900	
Tax			(180,000)	(186,750)	(174,488)	129,533
NCF after tax	(2,000,000)	900,000	610,000	482,250	381,413	129,533
DF	1	0.909	0.826	0.751	0.683	0.621
PV	(2,000,000)	818,100	503,860	362,170	260,505	80,440
NPV	25,074					

In this case, tax is payable one year after the profit to which it is related. The first year's tax is payable at the end of year 2 and the second year's tax is payable at the end of year 3 and so on. Despite this being a four-year project it now has cash flow (tax savings) arising in year 5. As we can see from Case 2, paying tax in arrears helps improve the after-tax NPV of the project. Consequently, the project should be accepted.

The timing of when tax is paid is therefore crucial for the evaluation of a project's acceptability.

Activity 2.4

Attempt Question 16, BMA Chapter 6.

See VLE for solution.

Replacement decision

When considering a scenario where we have to select mutually exclusive projects with different life spans and where each project can be replicated in exact cash flow patterns, the simple NPV rule might not necessarily give the correct advice. To see why this might be the case, let us consider the following example.

Example 2.7

Lion plc is considering two different machines in an operation. The following net operating cash outflows for these two machines are given:

$	**Year**				
Machines	0	1	2	3	4
A	(100,000)	(10,000)	(10,000)	(10,000)	(10,000)
B	(75,000)	(15,000)	(15,000)	(15,000)	

In this example, both machines have different life spans and cash flow patterns. How do we compare the value of using these two machines in the operations?

Suppose Lion plc has a cost of capital of 10% per annum. The NPV of running these two machines can be calculated as follows:

$	Year				
Machine A	0	1	2	3	4
NCF	(100,000)	(10,000)	(10,000)	(10,000)	(10,000)
DF	1	0.909	0.826	0.751	0.683
PV	(100,000)	(9,090)	(8,260)	(7,510)	(6,830)
NPV	(131,690)				

$	Year			
Machine B	0	1	2	3
NCF	(75,000)	(15,000)	(15,000)	(15,000)
DF	1	0.909	0.826	0.751
PV	(75,000)	(13,635)	(12,390)	(11,265)
NPV	(112,290)			

On the basis of the NPV calculations, it seems to cost the company less to run Machine B ($112,290 compared to $131,690). However, if the operation is a going concern and we have to replace the machine once it has expired, how do we know if Machine B still gives the best value to the company?

To answer this question, we need to find a way to compare the two machines' cash flows in a consistent manner. This can be done by converting a project's NPV into its annual equivalent value (AEV).

Suppose we can hire a machine, C, for $*x* each year for the next four years. It has the same functionalities as Machine A and the hiring company is responsible for all the running cost of Machine C. What would be the equivalent hiring cost we would be willing to pay if both machines (A and C) have the same value to the company?

For these two machines to have the same value to the company, their total running costs (measured at today's value, i.e. present value) must be identical. Consequently this means:

$$\frac{x}{1.1}+\frac{x}{1.1^2}+\frac{x}{1.1^3}+\frac{x}{1.1^4}=131{,}690$$

$$x\left(\frac{1}{1.1}+\frac{1}{1.1^2}+\frac{1}{1.1^3}+\frac{1}{1.1^4}\right)=131{,}690$$

$$x\left(A_{10\%,4\,years}\right)=131{,}690$$

$$x=\frac{131{,}690}{A_{10\%,4\,years}}=\frac{131{,}690}{3.169}=41{,}556$$

where $A_{10\%,\ 4\ years}$ is the annuity factor at 10% for 4 years

If we are indifferent between paying the annual hiring cost of $41,556 for Machine C and paying the purchase cost and annual running costs for Machine A, then the hiring cost of $41,556 must be the annual equivalent value of Machine A. From the above calculation, we can define the annual equivalent value of a project as:

$$\text{AEV} = \frac{\text{Project's NPV}}{\text{Annuity for the duration of the project}}$$

We can now convert Machine B's NPV into its AEV in the same way as the calculation above:

$$\text{Machine B's AEV} = \frac{112{,}290}{2.486} = 45{,}169$$

As long as Machines A and B have identical risk to the company, it would be more advantageous for Lion plc to invest in Machine A since it has a lower annual cost than Machine B.

However, we need to apply AEV in project appraisal with care. The comparison of two projects' AEV is only valid provided that:

- Projects can be replicated in exactly the same cash flow patterns whenever they expire.
- Projects have similar risk to the company.
- Technological changes are unlikely to affect the efficiency of either project.
- The expiration of the project will be many years hence (in theory, infinitely).

If these conditions are not met, then AEV would not be a sensible method to determine the replacement policy.

Delaying projects

In some cases it might be more advantageous for a company to delay the commencement of a project. This might be a result of one of the following:

- There might be uncertainty about the outcomes of the project. Delaying its commencement might allow the company to obtain vital information to revise future cash flows which might give a higher NPV.
- There might be capital rationing in the current period and the company needs to postpone the project due to shortage of funding.

In deciding whether a project should be postponed, we can treat the delay of each project as separate and mutually exclusive. We can then evaluate each option's NPV accordingly.

Example 2.8

Rooster Ltd. is considering a new product, a roast chicken stand. It allows a chicken to be roasted on all sides while maintaining its juiciness. The production requires a new machine which has a purchase price of $100,000 with four years of economic life and no residual value by the end of the fourth year. Each unit of the roast chicken stand is expected to generate a net contribution of $5 (selling price minus variable costs). Market research, which costs $25,000, indicates that future demand will be subject to the state of the economy. If the economy is strong, the demand will be 10,000 units per year for the next five years. However, if the economy is weak, the demand will fall to only 5,000 units per annum. There is an equal chance for each state of the economy to materialise. It is also expected that once the state of the economy is set, it will stay that way for the next four years.

Rooster has a choice to delay the production until the end of the year. If it does so, the whole production cycle will be shortened to three years. The same machine will still be required by the end of the year at the same expected purchase price. However, it can be sold for $25,000 at the end of the production process (i.e. three years after the commencement of production). But more importantly, delaying the commencement of production will allow the company to know exactly which way the economy is going to unfold for the next few years.

Advise the management on what action should be taken regarding this project.

Approach:

Introducing probability theory we can calculate the expected net present value, E(NPV), for the project.

If Rooster Ltd. commences the production now:

Expected demand in the next 4 years = 50% × 5,000 units + 50% × 10,000 units = 7,500 units

Contribution per year = 7,500 units × $5 = $37,500

	Year				
No delay	**0**	**1**	**2**	**3**	**4**
Machine	(100,000)				
Contribution		37,500	37,500	37,500	37,500
E(NCF)	(100,000)	37,500	37,500	37,500	37,500
DF	1	0.909	0.826	0.751	0.683
E(PV)	(100,000)	34,088	30,975	28,163	25,613
E(NPV)	18,838				

This is the expected NPV that the production could generate. However, the economy is weak, Rooster can only sell 5,000 units per year. What, then, would be the outcome of this state?

If Rooster is to face a weak economy for the next four years, the revised NPV will be as follows:

	Year				
Weak state	**0**	**1**	**2**	**3**	**4**
Machine	(100,000)				
Contribution		25,000	25,000	25,000	25,000
NCF	(100,000)	25,000	25,000	25,000	25,000
DF	1	0.909	0.826	0.751	0.683
PV	(100,000)	22,725	20,650	18,775	17,075
NPV	(20,775)				

In other words, there is a 50% chance that Rooster will suffer a negative NPV of $20,775. (If the good state had occurred at the outset then the NPV would have been $58,450. (NB. 0.5 × $58,450 + 0.5 × ($20,775) = $18,838.) Should the company delay the project and wait for the economic situation to materialise before committing to production? If the company delays the production by a year, there are two possible actions that the company will take by the end of the year. It could abandon production if a weak economy materialises. It would not be advantageous to produce if the company could only sell 5,000 units per year in a weak economy. You can check the NPV under this option. However, if a strong economy materialises in year 1, the company will commence production in that year with the following NPV:

	Year				
Delay	**0**	**1**	**2**	**3**	**4**
Machine		(100,000)			25,000
Contribution			50,000	50,000	50,000
NCF	0	(100,000)	50,000	50,000	75,000
DF	1	0.909	0.826	0.751	0.683
PV	0	(90,900)	41,300	37,550	51,225
NPV	39,175				

The expected NPV of delaying production would then be $19,587.5 (50% × 0 + 50% × $39,175). On the basis of the NPV consideration, it seems to be more advantageous for the company to delay production by one year.

In this example, deferring the project allows the company to eliminate the possibility of facing a loss in a weak economy. Even though the financial return

to delay the project seems low ($19,587.5 vs. $18,838), the risk elimination might be treated as more valuable by a more risk-averse company.

Sensitivity analysis

This method evaluates the impact of changes in a project's variables on its NPV. In a single variable situation, we can assess by how much a variable needs to change before a project returns a loss. For example, referring to the data in Example 2.4, we can ask:

1. By how much does the selling price need to drop before the project's NPV disappears?
2. By how much does the production cost per unit need to rise before the project's NPV disappears?
3. By how much does the discount rate need to rise before the project's NPV disappears?

To answer any of the above questions, we can use a trial-and-error method. With the aid of a spreadsheet and changing the parameters accordingly, we can see how the NPV will change. See the spreadsheet on the VLE of the demonstration.

Alternatively, we can observe the relationship of the variable with the overall NPV. Recall from Example 2.4 that each unit of the product sells at $200 and the NPV of the project stands at $865,760. Let's assume that the selling price of each unit drops by $x. To make the NPV disappear, the present value of the loss in revenue (or contribution) must be identical to the NPV. We, therefore, can equate the PV of the loss in contribution to the project's original NPV as follows:

$$\frac{10{,}000x}{1.1}+\frac{10{,}000x}{1.1^2}+\frac{10{,}000x}{1.1^3}+\frac{10{,}000x}{1.1^4}=865{,}760$$

$$10{,}000x \times A_{10\%,4}=865{,}760$$

$$10{,}000x \times 3.169=865{,}760$$

$$x=27.32$$

If the selling price drops by $27.32, the NPV will disappear. This gives a safety net for the company as to how much it can afford to reduce the selling price before incurring a loss. You can test each variable using the approach outlined above and determine how sensitive the NPV is to each of the variables considered. This is of benefit to management in both the decision-making phase and the project management phase.

Practical consideration

Graham and Harvey (2001) surveyed 392 chief financial officers (CFOs) in the USA. They asked each CFO to rank the importance of each appraisal method in practice. Figure 2.2 below shows the findings of their survey. Watson and Head (2010) summarise the findings as follow:

- Discounted cash flow methods appear to be more popular than non-DCF methods.
- Payback is used in large organisations in conjunction with other investment appraisal methods.
- IRR is more popular than NPV in small companies but NPV is the most popular investment appraisal method in large companies.
- ARR, the least popular investment appraisal method, continues to be used with other methods.

- Companies tend not to use sophisticated methods to account for project risk.
- Most companies allow for inflation when considering projects' future cash flows.
- Almost all companies use sensitivity analysis, an increasing minority of companies use profitability analysis, very few companies use the capital asset pricing model.

Appraisal technique	Popularity, % always or almost always
Internal rate of return	75.61
Net present value	74.93
Payback period	56.74
Sensitivity analysis	51.54
Discounted payback period	29.45
Accounting rate of return	20.29
Profitability index	11.87

Table 2.1: Popularity of evaluation techniques

Source: Graham and Harvey (2001)

A reminder of your learning outcomes

Having completed this chapter, as well as the Essential readings and activities, you should be able to:

- describe the commonly used investment appraisal techniques
- apply the discounted cash flow techniques in complex scenarios
- evaluate the investment decision process.

Practice questions

1. BMA Chapter 5, Questions 10–15.
   BMA Chapter 6, Questions 22, 26, 28 and 29.

Sample examination questions

1. Rabbit Inc. is considering the production of Product X and Y.

   Product X

   It can only be produced on a new machine, which has an expected cost of $200,000 and a four-year life span. The annual cash savings are expected to be $50,000 in the first year, rising at 20% per annum thereafter until the end of the production. The new machine will attract a capital allowance of 25% on the written down value of the machine in each year. The company can claim any unrelieved capital allowance at the end of the production.

   Product Y

   Production of Product Y is expected to last for three years. Sales are expected to be 1,000 units in the first year, 1,200 units in the second year, and 800 units in the third year. Each unit can be sold for $20.

   It can be produced on an existing machine which has been idle for some time. This existing machine can be sold immediately for $10,000. If the production does go ahead, a one-off modification on the machine will be needed at a cost of $15,000 payable at the beginning of the first year.

Each unit of Product Y requires 1 kg of material at $4 per kg and one hour of skilled labour at $4 per hour. These costs are expected to rise in line with inflation.

The company has a choice to defer the production of Product Y until the beginning of the second year. If the company defers this production, the first year sales contribution will be lost irrevocably.

The company also has an option to purchase a brand new machine for the production of Product Y. It will cost the company $50,000 now or $30,000 in one year's time. This machine does not qualify for capital allowance.

The company's policy is to depreciate machines over their useful economic life on a straight-line basis. No machine is expected to have any value at the end of its life.

The inflation rate is expected to be 5% per annum. The company's after-tax cost of capital is 10% per annum. Corporate tax rate is 30%, payable one year in arrears. Apart from the cash flows mentioned above, the company can raise an additional fund of $190,000 only at the beginning of year 1. There is no capital restriction in subsequent years.

Required:

Advise Rabbit Inc. of the best investment plan in the above situation.

2. Assume that you have been appointed as the finance director of Dragon plc. The company is considering investing in the production of an electronic security device, with an expected market life of five years.

   The previous finance director has undertaken an analysis of the proposed project; the main features of his analysis are shown below. He has recommended that the project should not be undertaken because the estimated annual accounting rate of return is only 12.3%.

	Proposed electronic security device project					
	Year 0 (£'000)	**Year 1 (£'000)**	**Year 2 (£'000)**	**Year 3 (£'000)**	**Year 4 (£'000)**	**Year 5 (£'000)**
Investment in depreciable fixed assets	4,500					
Cumulative investment in working capital	300	400	500	600	700	700
Sales		3,500	4,900	5,320	5,740	5,320
Materials		535	750	900	1,050	900
Labour		1,070	1,500	1,800	2,100	1,800
Overhead		50	100	100	100	100
Interest		576	576	576	576	576
Depreciation		900	900	900	900	900
		3,131	3,826	4,276	4,276	4,276
Taxable profit		369	1,074	1,044	1,014	1,044
Taxation		129	376	365	355	365
Profit after tax		240	698	679	659	679

Total initial investment is £4,800,000.

Average annual after-tax profit is £591,000.

All the above cash flow and profit estimates have been prepared

in terms of present day costs and prices (i.e. no inflation), since the previous finance director assumed that the sales price could be increased to compensate for any increase in costs.

You have available the following additional information:

a. Selling prices, working capital requirements and overhead expenses are expected to increase by 5% per year.

b. Material costs and labour costs are expected to increase by 10% per year.

c. Capital allowances (tax depreciation) are allowable for taxation purposes against profits at 25% per year on a reducing balance basis.

d. Taxation on profits is at a rate of 35%, payable one year in arrears.

e. The fixed assets have no expected salvage value at the end of five years.

f. The company's real after-tax weighted average cost of capital is estimated to be 8% per year, and nominal after-tax weighted average cost of capital 15% per year.

Assume that all receipts and payments arise at the end of the year to which they relate, except those in year 0, which occur immediately.

Required:

a. Estimate the net present value of the proposed project. State clearly any assumptions that you make.

b. Calculate by how much the discount rate would have to change to result in a net present value of approximately zero.

c. Compare and contrast the NPV and IRR approaches to investment appraisal.

Chapter 3: Risk and return

Essential reading

Brealey, R.A., S.C. Myers and F. Allen *Principles of corporate finance*. (New York: McGraw-Hill, 2010) tenth edition [ISBN 9780071314268] Chapters 7 and 8.

Further reading

Arnold, G. *Corporate financial management*. (Harlow: Financial Times/Prentice Hall, 2008) fourth edition [ISBN 9780273719069] Chapters 6–8.

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Aims

In this chapter we formally examine the concept and measurement of risk and return. In particular, we look at the necessary conditions for risk diversification, the portfolio theory and the two fund separation theorem. Asset pricing models are also discussed and practical considerations in estimating beta will be covered. Empirical evidence for and against the asset pricing models will be illustrated.

Learning outcomes

By the end of this chapter, and having completed the Essential reading and activities, you should be able to:

- describe the meaning of risk and return
- calculate the risk and return of a single security
- discuss the concept of risk reduction/diversification and how it relates to portfolio management
- calculate the risk and return of a portfolio of securities
- discuss the implications of the capital market line (CML)
- discuss the theoretical foundation and empirical evidence of the capital asset pricing model (CAPM) and its application in practice.

Overview

In Chapter 1, we mentioned that one of the key concepts in financial management is the relationship between risk and return. So how does this concept link to the value creation and project appraisal? In Chapter 2, we discussed the selection of suitable investment projects that would create value for a firm and its shareholders. We assume that those projects' cash flows are given with certainty. However, in reality, cash flows from an investment project seldom materialise as expected. So how might the variation of the realised cash flows affect an investment's value?

To be able to answer these questions, we will first need to understand what we mean by risk and how corporate managers can measure such risk.

Introduction of risk measurement

What is risk? For the purpose of financial management, risk is defined as the deviation of realised return from its expectation. If the returns of an investment follow a normal distribution, then risk can be proxied by its standard deviation. Mathematically, we denote this as:

$$Risk = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(R_t - \overline{R}\right)^2} \qquad (3.1)$$

where R_t is the return of an investment at time t (1 to T) and $\overline{R}$ is the mean return.

Example 3.1

Suppose we have an investment that has the following historic returns:

Year	Return (%)
1	10
2	–2
3	0
4	5
5	7

What is the risk of this investment?

Answer:

This investment has an average return of 4% for the last five years. We can assume that the historic mean return of this investment is 4% (our best estimate in the absence of any further information about this investment). We can calculate the standard deviation of the returns by taking the square root of the average of the squared deviation of each annual return to its mean. The following table lists the results.

Year	Return (%)	Deviation = Return – Mean Return	Squared deviation
1	10	6	36
2	–2	–6	36
3	0	–4	16
4	5	1	1
5	7	3	9
Sum	20		98
Average	4		24.5
Std dev			4.95

Based on the calculation above, we can say that this investment on average provides a return of 4% per annum. However, none of the past five years' returns meet the expected return. In years 1, 4 and 5, the realised returns are higher than expected (upside risk). Whereas in years 2 and 3, returns are lower than expected (downside risk). This kind of deviation from the mean (expectation) constitutes the concept of risk in financial management. As long as (i) the returns of an investment follow the normal distribution and (ii) investors have no preference toward the upside and downside risks, standard deviation will be a neat measurement of risk for this type of investment.

Activity 3.1

Attempt Question 3 of BMA, Chapter 7.

See VLE for solution.

Implications of using standard deviation as a risk measure

Using standard deviation as a proxy for risk allows us to explore the statistical property of a given investment. It enables us to estimate the probability of obtaining a particular return. Take a look at the following example.

Example 3.2

a. In Example 3.1, what is the probability that an investor will receive a return of 10% from the investment?

b. What is the probability that an investor will not suffer a loss in the investment?

Answers

a. The probability of a outcome from a normal distribution can be expressed as:

$$\text{Prob}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (3.2)$$

where μ is the mean, σ is the standard deviation and x is the outcome of the function.

Given in Example 3.1 that we have $x = 10\%$, $\mu = 4\%$ and $\sigma = 4.95$, we have:

$$Prob(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2 \times 3.1416 \times 24.5}} e^{-\frac{(10-4)^2}{2 \times 24.5}}$$

$$= 0.038$$

b. The probability for an investor not suffering from a loss is equal to the probability that the return is equal to or larger than 0%. Given that a normal distribution curve is symmetrical at the mean value, we can easily see that the probability of returns equal to and larger than 4% would be 50%. So what is the probability that a return is between 0% and 4%?

To answer this, we first define the *z*-value as:

$$z = \left| \frac{x - \mu}{\sigma} \right|$$

In this example, $z = (0 - 4)/4.95 = 0.808$.

Using the table 'Area under the standardised normal distribution'[1], we can determine the probability of return between 0% and 4% as 0.291.

Therefore the probability for an investor not suffering from a loss in this case would be equal to 0.791 (0.5 + 0.291).

[1] *This table can be found in Arnold (2008), Appendix 5.*

Activity 3.2

What would be the probability for an investor to earn a return of 8% in Example 3.1?

See VLE for solution.

Diversification of risk and portfolio theory

What can we do about the risk of an investment?

If we put all our money in one single investment, we will be facing the entire risk of that investment. Consequently, if the investment turns out to be loss-making, there is not much we can do about it. However, if we spread our money across different investments in the first place, there might be a chance that one investment's loss is compensated by another investment's gain. The overall variation of the realised return on a portfolio of investments might therefore be reduced. This is the concept of risk diversification.

Markowitz (1952) argues that combining different investments in a portfolio can reduce the overall standard deviation below the level obtained from a simple weighted average calculation provided that the investments are not all positively correlated. In general, the expected return on a portfolio with *N* investments and its variance can be expressed as follows:

$$E(R_p) = \sum_{i=1}^{N} \omega_i E(R_i) \qquad (3.3a)$$

and

$$\sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + \sum_{i \neq j}^{N} \omega_i \omega_j \sigma_{ij} \qquad (3.3b)$$

where $E(.)$ is the expectation function, ω_i is the weight in investment i, σ_{ij} is the covariance of returns between investment i and j.

Two-asset portfolio

Let's first examine how the risk of a portfolio with two securities can be calculated.

Example 3.3

Suppose that you are considering an investment portfolio with two stocks, Rose Plc and Thorn Plc. The returns of these two stocks for the last five years are in columns 1 and 2 of the table below.

	1	2	3	4	5	6	7
Year	Rose, R_x	Thorn, R_y	$R_x–E(R_x)$	$[R_x – E(R_x)]^2$	$R_y–E(R_y)$	$[R_y–E(R_y)]^2$	$[R_x–E(R_x)][R_y– E(R_y)]$
1	4	2	0	0	–1	1	0
2	11	–2	7	49	–5	25	–35
3	13	6	9	81	3	9	27
4	–8	–1	–12	144	–4	16	48
5	0	10	–4	16	7	49	–28
Sum	20	15		290		100	12
Mean	4	3	Variance	72.5	Variance	25	Covariance = 3
Standard deviation				8.5		5	
Coefficient of correlation							0.07

Note that the variance and covariance are calculated using the following formulas:

$$\sigma_x^2 = \sum_{t=1}^{T} \frac{\left[E(R_x) - \overline{R}_x\right]^2}{T-1} \quad (3.4a)$$

$$\sigma_y^2 = \sum_{t=1}^{T} \frac{\left[E(R_y) - \overline{R}_y\right]^2}{T-1} \quad (3.4b)$$

$$\sigma_{xy} = \sum_{t=1}^{T} \frac{\left[E(R_x) - \overline{R}_x\right]\left[E(R_y) - \overline{R}_y\right]}{T-1} \quad (3.4c)$$

To see the diversification effect, we first calculate the standard deviation of the two companies and their covariance. Covariance measures the co-movement of the two stocks. At first glance, Rose and Thorn are not moving in the same direction all the time, suggesting that they are not perfectly correlated. To see the extent of their co-movement, we compute the covariance and coefficient of correlation.

Next, we combine the two stocks with different weights in a portfolio. Using equations 3.3a and 3.3b we can compute the portfolio's risk and expected return based on different weights as follows:

Weight in Rose	Weight in Thorn	Portfolio's risk	Portfolio's *E*(*Rp*)
1	0	8.5	4
0.9	0.1	7.7	3.9
0.8	0.2	7	3.8
0.7	0.3	6.2	3.7
0.6	0.4	5.6	3.6
0.5	0.5	5.1	3.5
0.4	0.6	4.7	3.4
0.3	0.7	4.5	3.3
0.2	0.8	4.5	3.2
0.1	0.9	4.6	3.1
0	1	5	3

The portfolio's expected return is simply the weighted average of the two companies' returns. The portfolio's risk is determined by using the general formula and we can see that it gradually decreases as the weight in the lower risk company (i.e. Thorn) increases. We should also note that the portfolio's risk falls below 5% when the weight in Thorn exceeds 0.5.

We could plot the portfolio's risk against the expected return at different weights (see Figure 3.1). The solid line represents the efficient frontier which consists of all efficient portfolios that can be formed between Rose and Thorn.[2] Investors can decide which composition they want to take.

[2] An efficient portfolio is one that maximises expected return for a given risk.

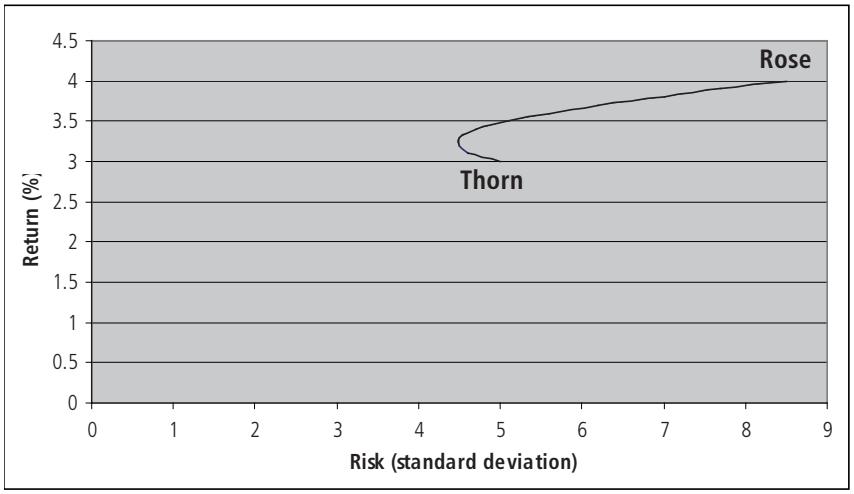

Figure 3.1: Efficient frontier of portfolios formed by two assets.

Activity 3.3

Suppose we have two stocks, A and B with the following characteristics:

	Return	Risk
A	10	10
B	5	5

Sketch the efficient frontier of a portfolio which consists of stock A and B, assuming that the coefficient of correlation equals:

a. 1

b. 0

c. –1

See VLE for solution.

Multi-asset portfolio

The above analysis can be extended to a multi-asset scenario. Suppose it is free to buy and sell assets to form a portfolio. An investor may want to combine more assets in her portfolio if more risk can be diversified. To see how this may work, let's take a look of the analysis below.

Recall equations 3.3a and 3.3b

$$E(R_p) = \sum_{i=1}^{N} \omega_i E(R_i)$$

$$\sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + \sum_{i \neq j}^{N} \omega_i \omega_j \sigma_{ij}$$

where σ_i^2 is the return variance of stock *i*

and σ_{ij} is the covariance between returns on stock *i* and *j*.

Suppose we put equal weight in each asset, the portfolio's risk will be reduced to

$$\sigma_p^2 = \frac{1}{N^2} \sum_{i=1}^{N} \sigma_i^2 + \frac{1}{N^2} \sum_{i \neq j} \sigma_{ij} \qquad (3.5a)$$

Define $\overline{\sigma^2}$ as the average variance and $\overline{\sigma_{ij}}$ as the average covariance,

$$\sigma_p^2 = \frac{1}{N} \overline{\sigma^2} + \left(1 - \frac{1}{N}\right) \overline{\sigma_{ij}} \qquad (3.5b)$$

If *N* is sufficiently large (i.e. $N \to \infty$), then

$$\frac{1}{N} \overline{\sigma^2} \to 0 \text{ and } \left(1 - \frac{1}{N}\right) \overline{\sigma_{ij}} \to \overline{\sigma_{ij}}$$

That implies

$$\sigma_p^2 = \overline{\sigma_{ij}} \qquad (3.5c)$$

The portfolio's risk is therefore determined by the average covariance among the stocks in the portfolio.

Implications

There are a few key implications from the above analysis worth noting.

i. As an investor combines more assets in a portfolio, the limiting portfolio's risk will gradually be reduced as both the first and second term in equation 3.5a will slowly disappear. Consequently, the shape of the efficient frontier will change and move more to the north-western quadrant of the mean-variance space.

   In Figure 3.2, each half-egg shell represents the possible weighted combinations for two assets. The composite of all assets constitutes the efficient frontier. The area underneath the efficient frontier consists of feasible but not efficient portfolios.

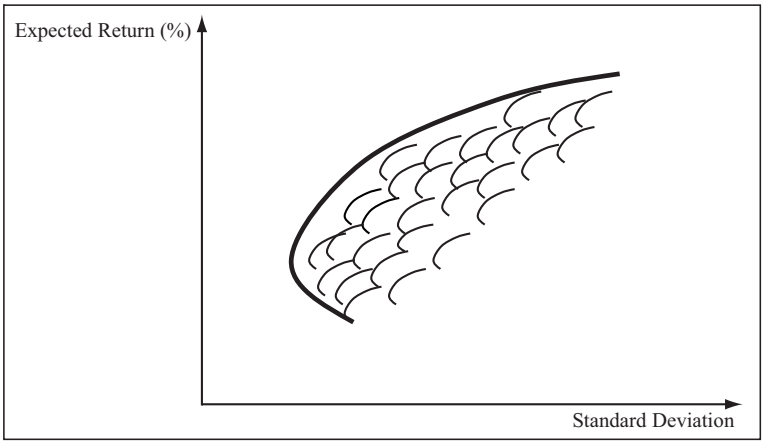

Figure 3.2: Efficient frontier of portfolios formed by multiple assets.

ii. The portfolio's risk will be minimised when a sufficiently large number of assets are included in the portfolio.

iii. If there is no transaction cost involved in portfolio forming, then a rational and sensible investor will combine all possible assets in her portfolio. Ultimately, the portfolio is composed of and reflects the entire market. The risk of such a portfolio must be the same as the risk of the market. Using equation 3.5c, the market's risk must be equal to the average covariance of the assets in the market:

$$\sigma^2_{market} = \overline{\sigma}_{ij}$$

iv. If one can lend or borrow at some risk-free rate of interest, an investor, who previously holds a risky portfolio on the efficient frontier, may now combine the risk-free asset with the market portfolio. This can be represented graphically:

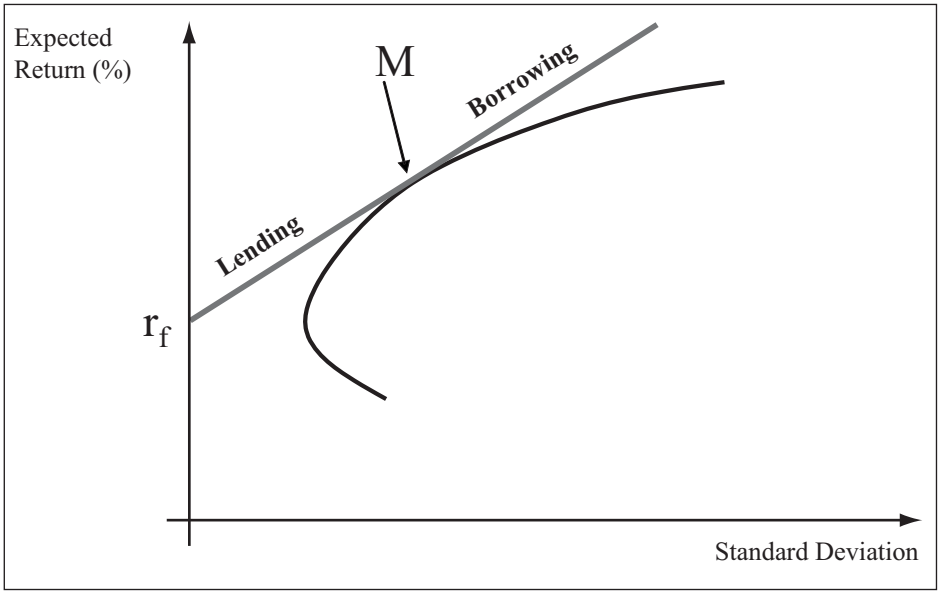

Figure 3.3: Efficient frontier and risk-free asset.

This is known as the two fund separation theorem. No matter what risk attitude an investor has, he or she will always seek to form a portfolio by combining the risk-free and a risky market portfolio on the efficient frontier. Given the two fund separation theorem and the implications of the portfolio theory, any efficient portfolio will lie on the capital market line (CML), which has the following form:

$$E(R_p) = R_f + \frac{E(R_m) - R_f}{\sigma_m}\sigma_p \qquad (3.6)$$

Activity 3.4

Equation 3.6 required that there is a single risk-free rate in the capital markets. In practice, investors could seldom borrow and lend at the same risk-free rate. How would this affect the capital market line?

See VLE for discussion.

Applications of the capital market line (CML)

The CML equation gives investors an idea of how much return should be expected for any given portfolio risk.

Example 3.4

It is expected that the market has an average return of 10% and the risk-free asset has a return of 5%. The standard deviation of returns on the market has been 7% in the past. What is the expected return of a portfolio with a standard deviation of 10%?

Using equation 3.6, we have

$$\begin{aligned} E(R_p) &= R_f + \frac{E(R_m) - R_f}{\sigma_m}\sigma_p \\ &= 5 + \frac{10-5}{7} \times 10 \\ &= 12.14 \end{aligned}$$

Activity 3.5

Attempt Question 5 of BMA, Chapter 8.

See VLE for solution.

In the above analysis, we address the issue of risk and return relating to a portfolio. We now turn our attention to individual assets. At the beginning of the chapter, we defined risk and return for a single investment. When an investor holds a single investment (or asset), he or she faces the entire variation of returns of that asset. Consequently, the standard deviation will be a good proxy for risk to such an investor. However as we have seen in the discussion of portfolio theory and diversification of risk, a sensible investor should form portfolios with many assets in order to eliminate 'risk'. The relationship between the number of assets and portfolio risk is depicted in Figure 3.4.

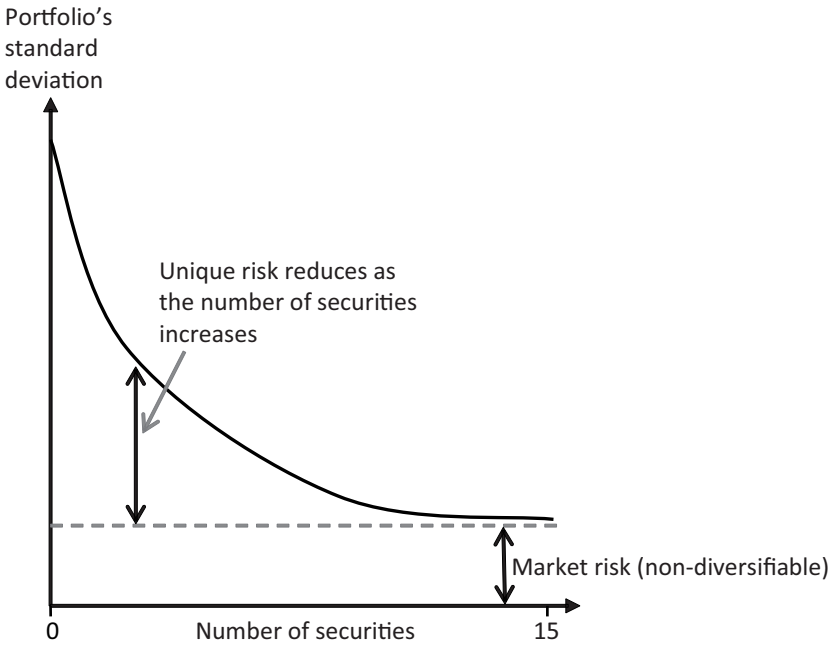

Figure 3.4: Risk diversification and the number of securities.

Figure 3.4 depicts two types of risk: risk that can be diversified (specific or unique risk) and risk that cannot be diversified (market or systematic risk). Empirically, an investor who holds 15 or more stocks in a portfolio would probably hedge most of the specific risk. Arguably the only risk in the portfolio would be from the market (the undiversifiable risk).

If transaction costs are negligible, investors would combine all assets in their portfolios. Ultimately, everyone will hold the same most diversified portfolio (the market portfolio). If this is the case, then how would investors price a new asset in the market?

Activity 3.6

Given Figure 3.4, what is the implication for small investors who have only a small amount of capital to invest?

See VLE for discussion.

Derivation of capital asset pricing model (CAPM)

BMA does not deal with the theoretical derivation for the capital asset pricing model. Here you can find a simple numerical derivation.

Suppose an investor holds a portfolio which combines *a%* of an asset *i* and (*1–a*)% of the market portfolio. The expected return and standard deviation of the portfolio *p* are

$$E(R_{pt}) = aE(R_{it}) + (1-a)E(R_{mt}) \tag{3.7}$$

and

$$\sigma(R_{pt}) = \left[a^2\sigma_i^2 + (1-a)^2\sigma_m^2 + 2a(1-a)\sigma_{im}\right]^{1/2} \tag{3.8}$$

where σ_i^2 is the variance of the return on the risky asset *i*; σ_m^2 is the variance of the return on the market portfolio; and σ_{im} is the covariance of returns between asset *i* and the market portfolio. The marginal rate of substitution (MRS) between the expected return and risk of the market portfolio is

defined as the ratio of the partial differentiation of its expected return over the partial differentiation of its expected risk of the portfolio with respect to *a*. In equilibrium, all marketable assets are included in the market portfolio and there is no excess demand or supply for any individual asset. This implies that:

$$\left.\frac{\partial E(R_{pt})/\partial a}{\partial E(\sigma_{pt})/\partial a}\right|_{a=0} = \frac{E(R_{it}) - E(R_{mt})}{(\sigma_{im} - \sigma_m^2)/\sigma_m} \tag{3.9}$$

Also note that the MRS at the point of the market portfolio on the efficient frontier is the same as the slope of the capital market line (CML) at the point of tangency to the efficient frontier. It can be shown that:

$$MRS = \frac{E(R_{it}) - E(R_{mt})}{(\sigma_{im} - \sigma_m^2)/\sigma_m} = \frac{E(R_{mt}) - R_{ft}}{\sigma_m} = Slope\ of\ the\ CML \tag{3.10}$$

Rearranging the equation 3.10, we have:

$$E(R_{it}) = R_{ft} + \beta_i[E(R_{mt}) - R_{ft}] \tag{3.11}$$

where $\beta_i = \sigma_{im} / \sigma_m$. Equation 3.11, also known as the equation of CAPM or security market line, shows that there is an exact linear relationship between an asset's return and its beta. This beta measures the risk of an asset relative to the market. We can from now on call it the market risk of an asset.

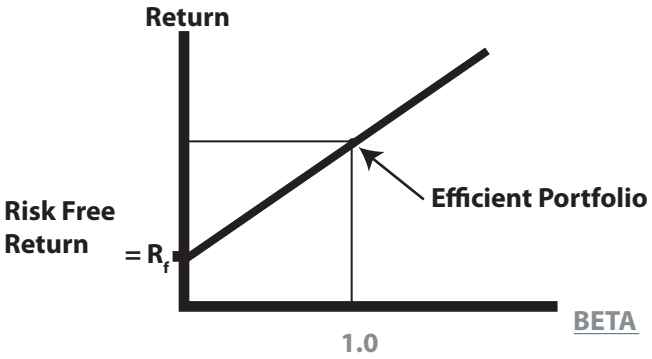

Figure 3.5: The CAPM and the security market line.

Activity 3.7

Attempt Question 7 of BMA, Chapter 8.

See VLE for solution.

Estimation of beta

Equation 3.11 depicts that there is a linear relationship between risk and return on individual assets. The risk is measured in terms of its risk relative to the market (beta) and return is what investors and the market would expect to receive given this level of market risk. Consequently knowing beta would allow us to estimate the expected return on an asset or security.

How do we estimate the beta for a company? The following example demonstrates a simple approach which can be used in practice.

Example 3.5

SpringTime plc is an all-equity financed company on the London Stock Exchange. For the last five years, its stock returns and the returns on FTSE100 are as follows:

Year	Returns on SpringTime (%)	Returns on FTSE100 (%)
1	10	8
2	6	1
3	-4	10
4	24	12
5	19	14

The risk-free rate is 5% per annum.

Using the above information, we can estimate beta and the expected return on SpringTime. The approach is as follows:

1. Compute the mean return of SpringTime and FTSE100 (Column I and II).
2. Determine the deviation of each observation from its mean (Column III and IV).
3. Calculate the covariance of returns between SpringTime and FTSE100 by averaging the sum of the products of each pair of deviations (Column V).
4. Calculate the variance of returns on FTSE100 (being the average of the sum of the squared deviations of Column VI).
5. Estimate the beta.
6. Substitute beta into the CAPM equation.

	I	II	III	IV	V	VI
Year	SpringTime's return	FTSE100's return	DEV, S	DEV, M	III × IV	IV × IV
	%	%	%	%	%	%
1	10	8	-1	-1	1	1
2	6	1	-5	-8	40	64
3	-4	10	-15	1	-15	1
4	24	12	13	3	39	9
5	19	14	8	5	40	25
Sum	55	45			105	100
Mean	11	9		Covariance	21	
				Variance		20

Beta is defined as the market risk of a company. This implies that we measure the covariance of the return relative to the risk of the market. In other words, beta can be calculated as follows:

$$Beta = \frac{Covariance}{Variance} = \frac{\sigma_{sm}}{\sigma_m^2} = \frac{21}{20} = 1.05$$

Substituting in the CAPM equation we have

$$E(R_s) = R_f + \beta_s\left[E(R_m) - R_f\right]$$

$= 5 + 1.05 \times (9 - 5)$

$= 9.2$

Activity 3.8

Attempt Question 15 of BMA, Chapter 8.

See VLE for solution.

There are a few aspects of estimating beta worth noting here:

i. In the absence of any information about the expected return on the market (i.e. the return on FTSE100 in the above example), the average historic return will be the only sensible proxy for the expected return.

ii. In the above analysis, we have not adjusted for the small sample error. If we do, the analysis will be as follows:

	I	II	III	IV	V	VI
Year	SpringTime's return	FTSE100's return	DEV, S	DEV, M	III × IV	IV × IV
	%	%	%	%	%	%
1	10	8	-1	-1	1	1
2	6	1	-5	-8	40	64
3	-4	10	-15	1	-15	1
4	24	12	13	3	39	9
5	19	14	8	5	40	25
Sum	55	45			105	100
Mean	11	9		Covariance	26.25	
				Variance		25

You can see that the differences lie on the calculation of the covariance and variance. We re-calculate the covariance and variance using equations 3.4a – 3.4c. However, the beta remains unchanged (26.25/25 = 1.05).

iii. The estimation of beta is sensitive to both the return intervals and the sample periods. Daves, Ehrhardt and Kunkel (2000) have the following conclusion:

> 'Financial managers can estimate the cost of equity via the CAPM approach. If the financial manager estimates the firm's beta... then the financial manager must select both the return interval and the estimation period. Regarding return interval... the financial manager should always select daily returns because daily returns result in the smallest standard error of beta or greatest precision of the beta estimate. However, regarding estimation period, the financial manager faces a dilemma. While a longer estimation period results in a tighter standard error for the estimate of beta, a longer estimation period also results in a higher likelihood that there will be a significant change in the beta. Thus, the beta estimated over longer estimation periods is more likely to be biased and of little use to the financial manager.'

Conceptual issues with CAPM

- The CAPM implies relationships between ex ante (expected) risk premia and betas that are not directly observable. However, in most empirical work, we implicitly assume that the realised returns on assets in a given time are drawn from the ex ante probability distribution of returns on those assets under rational expectations.

- Empirical tests use time-series data to calculate mean excess rates of return and betas; however, it is unlikely that risk premia and betas of individual assets are stationary over time. This issue is addressed by explicitly forming portfolios that are stationary over time or conditioning risk premia and betas on information sets.
- Many assets are not marketable and tests of the CAPM are inevitably based on proxies for the market portfolio. The test of the CAPM becomes a test whether the proxy is mean-variance efficient (Roll, 1977).

Empirical evidence and evaluation

Both BMA Chapter 8 (pp.223–27) and Arnold Chapter 8 (pp.295–99) discuss the empirical evidence of the CAPM. The key question one must ask is whether the CAPM is indeed the true model for explaining the risk and return of individual companies. The summary of empirical evidence from the textbooks and the text given in this section both indicate that the cross-sectional returns are not explained solely by the market risk factor, beta. There appear to be other risk factors which explain the expected returns of individual companies.

It is advised that you read and make notes from BMA Chapter 8 (pp.223–27) and Arnold Chapter 8 (pp.295–99) before proceeding with the rest of this section.

An extensive body of empirical research has provided evidence contradicting the prediction of the CAPM. This research documents that deviations from the linear CAPM risk-return trade-off are related to, among other variables, firm size (defined as the natural logarithm of the market value of a firm), earnings yield (defined as the earnings per share of a firm over its share price value), leverage (measured as the ratio of debt to equity) and the book-to-market ratio (defined as the net book value of a firm over its market value). Ferson (1992) provides an extensive summary of these empirical tests on the CAPM and its anomalies up to 1991. Other notable works include:

Banz (1981), Basu (1983), and Reinganum (1981) show that the firm size and earnings yield can explain the cross-sectional returns in conjunction with the market beta, suggesting that beta is not the only risk factor.

Fama and French (1992) show that by employing a new approach for portfolio grouping, there is only a weak positive relation between average monthly stock returns and market betas over the period from 1941 to 1990. This relation virtually disappears over a shorter period from 1963 to 1990. However, firm size and the book-to-market equity ratio have considerable power. The findings in this paper cast serious doubt over the validity of the CAPM as the true cross-sectional asset pricing model and has stirred up a new wave of empirical attention on the CAPM.

However in any CAPM test, there are two issues that need to be resolved:

i. Is the data for measuring or testing the expected returns taken from a complete set which has no bias? Kothari, Shanken and Sloan (1995) argued that if portfolios are formed from a data set which contains only the surviving firms, the CAPM test might not be conclusive.

ii. How can we be sure that the betas were correctly estimated in Fama and French (1992) if significant estimation errors are found in the estimated betas? If such estimation errors exist, then the tests on the significance of the betas in cross-section regression would be undermined (Kim, 1995).

Debate about whether the CAPM is the true pricing model is still going on.

The following section outlines the alternative pricing models.

Alternative asset pricing models

BMA Chapter 8 (pp.227–31) discusses the alternative pricing models. Read this section in the textbook before continuing with this section.

Multifactor models can be divided into three types: statistical, macroeconomic and fundamental factor models. Statistical factor models derive their pervasive factors from factor analysis of the panel data set of security returns. Macroeconomic factor models use observable economic time series, such as inflation and interest rates, as measures of the pervasive shocks to security returns. Fundamental factor models use the returns on portfolios associated with observed security attributes such as dividends yield, the book-to-market ratio, and industry identifiers.

The basic model takes the following form:

$$E\left(r - r_f\right) = b_1\left(r_{factor\,1} - r_f\right) + b_2\left(r_{factor\,2} - r_f\right) + ...$$

One of the most cited literatures on macroeconomic variable model is Chen, Roll and Ross (1986). They observe that asset prices are driven by exogenous forces as daily experience seems to support the view that prices react to unexpected news. If these forces are not diversifiable, the market will compensate investors for bearing those risks. They find that, most notably, the industrial production, changes in the risk premium, twists in the yield curve, and somewhat more weakly, measures of unanticipated inflation and changes in expected inflation during periods when these variables were highly volatile are significant. Contrary to previous belief, they find that the market return is not priced despite its high content of explanatory power.

Fama and French's three factor model is a good example of a fundamental factor model. BMA Chapter 8 (pp.229–31) explains how this model can be estimated.

Activity 3.9

Attempt Question 21 of BMA, Chapter 8.

See VLE for solution.

Practical consideration of CAPM

Despite the highly unrealistic assumption underpinning the CAPM and the empirical evidence against the model, practitioners remain faithful to the CAPM. Harvey and Graham (2002) examine what CFO's use in practice to estimate the cost of capital for their companies. Over 70% of the respondents rank the CAPM as the most popular tool to estimate the cost of capital.

Estimation method for cost of capital	Popularity, % always or almost always
CAPM	73.49
Average historic return	39.41
CAPM with extra risk factors	34.29
Discounted dividend model	15.74
Investors' expectation	13.93
Regulatory decisions	7.04

Table 3.1: Popularity of methods of calculating the cost of equity capital.

Source: Graham and Harvey (2001)

A reminder of your learning outcomes

Having completed this chapter, as well as the Essential reading and activities, you should be able to:

- describe the meaning of risk and return
- calculate the risk and return of a single security
- discuss the concept of risk reduction/diversification and how it relates to portfolio management
- calculate the risk and return of a portfolio of securities
- discuss the implications of the capital market line (CML)
- discuss the theoretical foundation and empirical evidence of the capital asset pricing model (CAPM) and its application in practice.

Practice questions

1. Suppose we have the following inflation rates, stock markets and US Treasury Bill returns between 2006 and 2010:

Year	Inflation (%)	S&P 500 Return (%)	T-bill Return (%)
2006	3.3	23.1	5.2
2007	1.7	33.4	5.3
2008	1.6	28.6	4.9
2009	2.7	21.0	4.7
2010	3.4	-9.1	5.9

   a. What was the real return on the S&P 500 in each year?

   b. What was the average real return?

   c. What was the risk premium in each year?

   d. What was the average risk premium?

   e. What was the standard deviation of the risk premium?

2. Is standard deviation an appropriate measure of risk for financial investments or projects? Discuss.

3. A game of chance offers the following odds and payoffs. Each play of the game costs £100, so the net profit per play is the payoff less £100:

Probability	Payoff	Net Profit
0.1	£500	£400
0.5	£100	0
0.4	£0	–£100

   What are the expected cash payoff and expected rate of return? Calculate the variance and standard deviation of this rate of return.

4. What do you understand by the term ‘risk and return’ in the context of financial management?

Sample examination questions

1. Suppose that you are considering investing in only two companies, Rose Plc and Thorn Plc. Their returns for the last five years are as follows:

Year	Return on Rose Plc (%)	Return on Thorn Plc (%)
–5	8	6
–4	20	7
–3	16	8
–2	4	–1
–1	12	10

Required:

   a. Calculate the expected return and standard deviation to the nearest percentage of both Rose and Thorn.

   b. Calculate the coefficient of correlation between the return of Rose and Thorn.

   c. Suppose you combine 50% of Rose and 50% of Thorn in a portfolio. Calculate the portfolio's expected return and standard deviation.

   d. Suppose that the risk-free rate is 6%. Explain, with the aid of a graph, the composition of an optimal portfolio.

2. James, who is a risk averse investor, is deciding how to divide his money between two assets, peppers and corn, which have the following characteristics:

	Peppers	Corn
Expected return	10	10
Standard deviation	5	5

   If the returns on Peppers are independent of those on Corn, what will be the composition of his optimal portfolio? Would the composition of the portfolio be different if a risk-free investment is available to James?

3. What are the necessary conditions for an efficient diversification of risk?

   a. What is the relationship between the number of available securities and the gains from diversification?

   b. Does this relationship have any implication for the small investor?

   c. 'In theory, an investor in risky securities is presumed to select an investment portfolio which is on the **efficient frontier** and touches one of his **indifference curves** at a tangent. But in practice, neither the efficient frontier nor the indifference can be estimated with high degree accuracy. Therefore, the **portfolio theory** is redundant.'

   Explain the terms in bold in the above statement. Critically assess their validity.

4. Suppose that you have estimated the expected returns and betas of the following five stocks using annual data available for the last 10 years:

Stock	Market Size (£m)	Beta	Expected Return (%)
A	360	0.6	8
B	23	0.8	10
C	250	1.2	11
D	10	1.3	12
E	500	1.4	12

The risk-free rate of interest and the expected return on the market are 5% and 10% respectively. You are also told that the market size of the companies in this market is normally distributed with a mean of £400m and a standard deviation of £150m.

Required:

a. Explain carefully the extent to which these data are consistent with the capital asset pricing model and whether there is any optimal investment strategy.

b. What would you advise an investor who would like to hold a portfolio with a beta equal to 1?

c. 'The risk of a company depends upon much more than how well the stock market is doing. Beta only captures the co-movement of a company's share price with the market; and hence fails to capture various sources of risk.'

Critically assess the validity of the capital asset pricing model and its use as a benchmark for project appraisals.